ROBUSTNESSVERSUSSTABILITYMEASURESINPROACTIVESCHEDULINGFORTIMETABLINGAPPLICATIONS

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Preface

Writing this master dissertation was a challenging and demanding experience, one that I couldn't have undertaken without the invaluable help of several individuals whom I would like to express my gratitude to in this preface. First of all, I would like to thank my supervisor Prof. Dr. Broos Maenhout for giving me the opportunity to learn and grow in the process of this dissertation. I would also like to thank Tessa Borgonjon for always being approachable for questions and helping me with the modeling and programming of the subjects in this dissertation.

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1 General introduction

Personnel scheduling and management have been studied extensively over the past decades, in part due to the economic benefits resulting from reducing labor costs by optimizing the personnel schedule (Ernst et al., 2004; Van den Bergh et al., 2013). Cutting even a small percentage of these direct costs can be very profitable for an organization. An optimized personnel roster not only leads to the minimization of costs but also to the optimization of the customer experience by matching demand and supply (Dorne & Dorne, 2008). The third objective of personnel scheduling is to maximize employee satisfaction (J. Bard and Purnomo, 2005; Moz and Pato, 2003; Topaloglu and Selim, 2010), since a company's performance depends largely on its employees.

Burke et al. (2004), Ernst et al. (2004), Dorne and Dorne (2008), Van den Bergh et al. (2013), and De Bruecker et al. (2015) all point out the cruciality of the personnel task scheduling problem because of its applicability in many different fields such as construction, transportation, manufacturing, hospitals and services in general. Nevertheless, the majority of the existing research conducted on this subject matter primarily concentrates on the development of precise and heuristic techniques for constructing a foundational schedule, assuming an environment characterized by determinism and complete information. However, the latest wave of economic globalization showed us once more that organizations operate in a dynamic and stochastic operational environment and face uncertainties on a daily basis.

Unpredicted events may make your schedule infeasible, meaning that it no longer satisfies all the relevant constraints and requirements. According to Van den Bergh et al. (2013), three sources of uncertainties in personnel scheduling can lead to the infeasibility of the schedule, namely the uncertainty of demand, the uncertainty of arrival, and the uncertainty of capacity. These indicate respectively the uncertainty of workload, arrival patterns of the workload, and resources available to satisfy the scheduled tasks.

The main strategies to minimize the impact of uncertainty on your schedule are proactive and reactive scheduling. Using the latter strategy involves revising and/or rescheduling when an unexpected event occurs. Alternatively, in proactive scheduling, one constructs predictive schedules that minimize disruptions' effect on the schedule's primary performance measure (Ghezail et al., 2010). In this way, some of the uncertainties are incorporated upfront when constructing the schedule.

The two primary performance measures of the schedule under investigation in this dissertation are robustness and stability. Both stability and robustness are concerned with the persistence, or lack thereof, of the specified features of a given system, with specified perturbations applied to the system (Jen, 2005). Robustness measures minimize disruptions' effect on the realized schedule's performance, while stability measures minimize the deviation from the original roster. Scheduling problems suffer from uncertainty and, therefore, more emphasis should be placed on robustness and stability, instead of solely on optimality in terms of cost minimization, employee satisfaction, and so forth (Golpıra & Khan, 2019). There has been limited research conducted regarding personnel scheduling with stability and robustness as objectives. The objectives pursued in the existing literature revolve around achieving a harmonious alignment between supply and demand, optimizing employee satisfaction, adhering to workplace constraints, and so forth. This dissertation will be based on the small part of the literature that does focus on strategies to improve the robustness and/or stability of personnel scheduling, especially the analysis of Ingels and Maenhout (2015), Ingels and Maenhout (2017), and Ingels and Maenhout (2018). In their research, they determine the impact of proactive and reactive strategies on the robustness of personnel rosters, by incorporating respectively reserve duties, employee substitutability, and overtime. However, the scope of this dissertation is to solely evaluate the impact of proactive strategies on robustness and stability. As such, the use and impact of reactive strategies will be excluded from the rostering process.

It is important to note here that the models Ingels and Maenhout use in their analysis applies to the personnel shift scheduling process. Since this dissertation investigates proactive strategies applied to the personnel task scheduling phase, slight adaptations were made to the models. A comparative analysis of these proactive strategies in terms of stability and robustness, especially in timetabling applications, has not yet been performed and is therefore the scope of this dissertation.

1.1 Content outline

The goal of this master's dissertation is to analyze and compare different types of proactive scheduling objectives for timetabling in terms of robustness and stability. The first part of this dissertation contains a literature review of the topic. The literature review consists of 2 chapters. The first chapter of this review, Chapter 2, gives an overview of the personnel scheduling process since this will be the domain of application of the different strategies. It also lists the three types of operational variability and the strategies to cope with these variabilities. Subsequently, Chapter 3 enumerates plural proactive robustness strategies for the personnel task scheduling problem.

The problem description, formulation, and methodology will be tackled in the second part. The problem description and methodology are covered in Chapters 4 and5. In the following chapter, Chapter 6, the baseline roster is formulated, which will be used in the simulation. In Chapters 7 and 8, the two chapters of the literature review are linked, namely by incorporating the different proactive strategies into the tactical phase of the personnel scheduling process. To do so, each proactive strategy is included in the baseline roster from Chapter 6.

The experimental research will be described in the third part of this dissertation. Given the multitude of application areas of the personnel scheduling problem, many exact and heuristic solution approaches have been introduced (Burke et al., 2004; Ernst et al., 2004; Van den Bergh et al., 2013). Most solution methodologies break this complex scheduling problem into multiple smaller problems, with their own constraints and objectives. In this dissertation, a baseline personnel roster is constructed in the integrated strategic staffing and tactical scheduling phase by making use of a branch-and-bound methodology. In the second phase, a Monte Carlo simulation is used to reproduce the uncertainty. The baseline schedule from the first phase will then be adjusted to the simulated variability and its performance will be assessed in terms of stability and robustness. Numerous other approaches to this problem exist; pre-emptive programming (Topaloglu & Ozkarahan, 2004), branch-and-bound (Trivedi & Warner, 1976), column generation (J. Bard & Purnomo, 2005) and branch-and-price (Ingels and Maenhout, 2015; Maenhout and Vanhoucke, 2010).

The objective of this simulation is to examine the relationship between robustness and stability. This part contains three big parts; a description of the simulation framework, the computational results, and a conclusion that puts everything together. The simulation framework is clarified in Chapter 10. The results of this simulation are described in the next chapter, Chapter 11. To wrap up this dissertation, Chapter 12 recapitulates the different conclusions and recommendations resulting from the simulation. The chapter also incorporates the limitations of this dissertation and the opportunities for future research.

Part I: Literature review

2 Personnel Scheduling

Personnel scheduling or personnel rostering is the process of establishing timetables for staff to fulfill its customers' requirements while respecting workplace agreements and satisfying individual work preferences (Ernst et al., 2004). Different objectives can be pursued when constructing an appropriate work schedule for your employees, such as minimizing personnel costs (Hazır et al., 2010; Koutsopoulos and Wilson, 1987; Stojković et al., 1998; Tam et al., 2011), maximizing productivity (J. Bard and Purnomo, 2005; Koutsopoulos and Wilson, 1987; Moreno-Camacho and Montoya-Torres, 2015), maximizing the personnel satisfaction (J. F. Bard and Purnomo, 2005; Pato and Moz, 2008; Topaloglu and Selim, 2010), etc. The objective of enhancing employee happiness by working part-time or working from home is present in every organization nowadays but makes the personnel scheduling process even more complex.

Not only do different objectives need to be kept in mind, but the timetable also needs to meet multiple (workplace) constraints. The maximum number of consecutive working days/hours and time between shifts are some examples hereof. Zucchi et al. (2021) recently added new criteria as a consequence of the Covid-19 pandemic, namely the risk of contagion among staff. This entails that the amount of co-workers your employees encounter physically needs to be reduced.

The process of personnel scheduling includes three hierarchical stages or phases; the strategic staffing phase, the tactical scheduling phase, and the operational allocation phase (Abernathy et al., 1973; Burke et al., 2004). Each phase has its time horizon and thus a different amount of information available at the moment that certain decisions need to be made. Consequently, decisions made in the earlier stages of the personnel scheduling process are based on assumptions about future events and put constraints on the decisions that will be made in later phases. In reality, the first two phases are combined into the integrated strategic staffing and tactical scheduling phase (Maenhout & Vanhoucke, 2013a). The different phases and their decisions are indicated in Figure 1.



Figure 1: The Personnel Scheduling Process

Strategic staffing phase

Budgeting, staffing, and short-term scheduling problems often arise when scheduling personnel. Even though these problems have different time horizons, they are all intertwined. The budgeting and staffing problems are (partly) dealt with in the first phase of the personnel planning process, namely in the strategic staffing phase. The personnel planner is faced with the trade-off between a large workforce, leading to declining productivity, and a small workforce, leading to high marginal costs due to overtime (Koutsopoulos and Wilson, 1987; Tan, 2003; Ingels and Maenhout, 2018). The personnel mix that is necessary to deliver the current and planned business objectives must be determined, meaning the settlement on the personnel mix and budget. Since the decision horizon in strategic staffing is the longest, there is less information about the future operating environment of the organization available. Because of this deficiency of information, the decisions in this phase are based on aggregated information, assumptions, and forecasts.

Tactical scheduling phase

The second phase of the personnel scheduling process is the tactical scheduling phase where the baseline personnel roster for a medium-term period is constructed. This phase is sometimes called the shift scheduling phase. An initial roster where staff is assigned to shifts and tasks is constructed based on the decisions made in the strategic staffing phase. The personnel planner is constrained by the strategic decisions made in the previous phase. The number and type of employees are limited by the personnel mix and budget. The extend to which proactive strategies can be included in the initial roster also depend on the decisions made before. But since the decision horizon is shorter, the planner possesses information about the amount of staff that will be available. This information is still subject to uncertainty, leading to supplementary assumptions.

Operational allocation phase

In the operational allocation phase the personnel roster is executed, as such the planner has up-to-date info about the operating environment. Adjustments must be made when the assumptions the personnel planner has made don't correspond with reality. Discrepancies of this kind are possible due to operational variability. Proactive and/or reactive strategies can be used to cope with these deviations from the baseline schedule.

The remainder of this chapter about personnel schedule is structured as follows. In Section 2.1 the rostering process is explained. As mentioned above, the personnel scheduling process does not happen in a deterministic environment. The operational variability that leads to the stochastic operation environment is discussed in Section 2.2. In the next section, Section 2.3, the two main types of strategies to minimize the impact of these uncertainties are explained. To conclude this chapter about personnel scheduling, robustness, and stability in personnel scheduling are explained in Section 2.4.

2.1 The rostering process

Ernst et al. (2004) represent the rostering process as several modules starting with the determination of staffing requirements and ending with the specification of the work to be performed over a period of time by each individual in the workforce. The procedures in the modules provide a general framework within which different rostering models and algorithms may be placed. The six different modules will be discussed in the six following sections. It is important to note here that these modules do not represent six sequential steps in the rostering process. Depending on the application some modules can be combined in one procedure and some other modules are reluctant. Since assignment can occur as a subproblem at various modules of the rostering process the three types of assignment are pointed out in Section 2.1.2

2.1.1 The process

Module 1: Demand modeling

The first module consists of the determination of the demand of staff at different times over a certain planning period, or rostering horizon. The demand modeling process is the process of translating some predicted pattern of incidents, a per shift specification of staff level for example, into associated duties. Then these duty requirements are used to establish a demand for staff. There are three broad incidents categories on which staff demand can be based: taskbased demand, flexible demand, and shift-based demand (Ernst et al., 2004). In task based demand, the demand is obtained from lists of individual tasks to be performed. These lists of tasks usually contain the time window in which the task must be completed, the starting time and duration, and the skills required to perform the task. Sometimes the task may be associated with a location. An initial demand modeling step is to combine individual tasks into task sequences that can be carried out by one person. Transportation applications, like the railway and airline industry, often make use of this demand modeling module.

In the case of flexible demand, forecasting techniques are used because the likelihood of incidents is less known. The outcome is usually a specification of the number of staff required at different times of the day for each day of the rostering period. There are two ways to handle this flexible demand once it has been determined for the whole rostering period (Ernst et al., 2004). The demand can be assigned to shifts, which can then be scheduled in lines of work. Alternatively, the demand may be fed directly into the later rostering phases as a constraint on the number of staff working at each time. In this case, penalties can be introduced when under – or overcovering the demand.

In shift-based demand, the demand is obtained directly from a specification of the number of staff that are required to be on duty during different shifts. It is mostly used in nurse scheduling applications where staff levels are determined from a need to meet service measures such as nurse/patient ratios and response times (Ernst et al., 2004).

Module 2: Days of scheduling

The constraint on the maximum number of consecutive working days/hours is dealt with in this module, since the rest days and how they need to be integrated into the roster are decided on in this step. This module is especially important when rostering to flexible and shift-based demand, it is of less relevance when rostering to task-based demand.

Module 3: Shift scheduling

There exist many possible shift patterns that can be implemented to meet demand, but in this module, one must decide on the set of shifts that fit the demand of employees the most and the number of employees for each shift. This step is redundant when rostering to shift-based demand, however, it needs extra attention when rostering to flexible and task-based demand. Namely, the timing of work and meal breaks need to be according to company requirements and workplace regulations when rostering to flexible demand. In task-based demand, the main goal is to select a good set of feasible duties, shifts, or pairing to cover all the tasks (Ernst et al., 2004).

Module 4: Line of work construction

In this module the complete work schedules, which are also called lines of work or roster lines, for each staff member over the rostering horizon are created. This process depends on the basic building blocks, typically shifts, duties, or stints, that are used (Ernst et al., 2004).

Any shift can be assigned to the workday of an individual if the basic building blocks are shifts. However one should keep in mind that some constraints need to be respected. These constraints limiting the valid shift pattern can for example be the regulation that no night shift may be followed by a day shift because of the maximum of consecutive working hours/days (Cheang et al., 2003). An aggregation of tasks during the demand modeling process or the workplace regulations and rules can lead to stints. These are predefined sequences of shifts and rest days.

Duties consist of (a sequence of) tasks that can be performed in only a part of the shift or even several shifts. Nevertheless, each duty can only be scheduled once in a roster. This aggregation of several tasks into one large piece of work is called duty generation and often occurs when dealing with task-based demand. In the next module, task assignment, these large pieces of work are seen as indivisible.

In this way, local dependencies are dealt with because multiple tasks, that require to be done at a specific location or different locations close by, can be combined in one trip. The main aims are to minimize paxing (the movement of crew between locations with no assigned duties during the transfer), time away from home, quality of life, etc. (Ernst et al., 2004).

The rules relating to the line of work, that ensure the feasibility of an individual's line of work, and the ones regarding the demand of work, that satisfy the work requirements in the rostering horizon, are considered at all times in this module. When rostering to flexible demand the module is usually called tour scheduling and crew rostering when dealing with crew pairing (Ernst et al., 2004).

Module 5: Task assignment

In the task assignment module, a set of tasks is assigned to the different shifts. These tasks may have a specified time window and particular staff skills or level of seniority and must therefore be assigned to particular work schedules. It is important to note here that it is not always necessary to assign tasks to shifts.

Module 6: Staff assignment

In this final module, each staff member is assigned to a line of work. This is mostly done during the construction of work lines. Even though it would be desirable to deal with all these modules simultaneously to generate the best overall rosters, it is common to break the planning problem into several separate modules to make it more tractable for the personnel planner. This decomposition also makes more sense in an organization's business practices. For example, the personnel planner can already decide on the shifts and the staff for those shifts, but delay the decision on which tasks to assign to those shifts when he/she has more information available.

2.1.2 Assignment

Assignment can occur as a subproblem at various steps in the rostering process. The most frequently used assignments are task assignments, shift assignments, and roster assignments (Ernst et al., 2004).

When working shifts have been determined but tasks have not been assigned to individual employees (module 3), task assignments are required. Based on their starting times and duration the tasks are grouped and assigned to staff or shifts. The methods for assignment depend on multiple criteria, namely on whether task times are fixed or movable, breaks exist in shifts, overtime is allowed, or specific skills or qualifications are required to perform the task (Ernst et al., 2004). In module 4, line of work construction, task assignment can also occur.

In module 4, line of work construction, shift assignments are used. Sequential assignment methods, like assigning the highest priority duties/pairings to the highest priority employees and assigning duties/pairings to employees on a day-to-day basis, are frequently applied to generate rosters.

Allocating rosters to individual employees can be done either after module 4 when all rosters have been created or during the construction of the lines of work. Individual staff preferences, availability, and qualifications can be included when roster assignment occurs during module 4, the construction of lines of work. A working pattern of an employee consists of tasks which the employee is qualified to perform, at time periods the employee is available (Brucker et al., 2011).

2.2 Operational variability

The introduction of this chapter about personnel scheduling already emphasized the problem that some decisions in the personnel scheduling process are made even though the personnel planner does not yet have all the required information available. Due to this stochastic operation environment, disruptions can make your schedule infeasible, meaning it no longer respects all relevant constraints and requirements. Van den Bergh et al. (2013) pointed out three types of uncertainty that lead to operational variability, i.e. uncertainty of demand (2.2.1), the uncertainty of arrival (2.2.2), and the uncertainty of capacity (2.2.3). The first and last types of uncertainties will be incorporated into the computational experiments of this dissertation. How these will be implemented in the simulation will be elaborated on in Chapter 9.

2.2.1 Uncertainty of demand

The unpredictability of workload is the result of the uncertainty of demand (Van den Bergh et al., 2013). If the encountered demand differs from the forecasted demand, the scheduled staff may need to be reduced or increased to meet this actual demand. This type of variability impacts the set of tasks that can be tackled in a specific shift. Examples hereof are the unpredictability of the number of patients entering the EER of a hospital, the number and duration of calls in a call center, etc.

2.2.2 Uncertainty of arrival

Not only the amount of demand is uncertain, but also the arrival pattern of the demand is unpredictable. This impacts the starting times, end times, and duration of the different tasks during the shifts. The personnel schedule needs to be adjusted because of the modification of the timing of the need for staff. The arrivals of calls in a call center or the distribution of failures over time of a machine (part) are some cases of the uncertainty of arrival.

2.2.3 Uncertainty of capacity

The third type of variability is the uncertainty of capacity, i.e. the difference between the planned and the actual manpower. The unavailability of a staff member, due to illness for example, leads to a lack of capacity. The set of workers you can assign to particular shifts is affected by the uncertainty of capacity.

2.3 Proactive and reactive personnel scheduling

In the first two phases of the personnel scheduling process, the strategic staffing and tactical scheduling phase, the personnel planner lacks some information needed to make the right operational decisions. To fill this gap of information, assumptions are made. In the last phase, the operational allocation phase, the baseline schedule is executed in real-time, meaning that now we get up-to-date information about the operational environment. Uncontrollable factors may cause the baseline schedule to no longer be feasible in the operational allocation phase.

The previous section, Chapter 2.2, enumerated the three types of operational variability that can lead to this infeasibility. In this chapter, two types of strategies to cope with these uncertainties are explained. Proactive scheduling is described in Section 2.3.1, this strategy is used to protect the baseline schedule against disruptions. Alternatively, the baseline schedule is re-optimized or re-vised when unexpected events occur in reactive scheduling (Section 2.3.2). The former, proactive strategies, are used in the offline operational scheduling phase, while reactive strategies are used in the online operational scheduling phase (Maenhout & Vanhoucke, 2013b).

2.3.1 Proactive Personnel Scheduling

In proactive scheduling one constructs predictive schedules that minimize the effect of disruptions on the performance measures of the schedule (Ghezail et al., 2010). To do so, some of the uncertainties are incorporated upfront when constructing the personnel roster. In general, proactive strategies aim to anticipate operational variability and unexpected events such as employee absenteeism. This results in improved stability and/or flexibility of the personnel roster.

The introduction of this chapter mentioned that the personnel mix and budget are determined in the strategic staffing phase and that decisions made in this first phase impact the decisions to be made in the second phase, the tactical scheduling phase. In this tactical scheduling phase, the personnel planner creates a timetable that is able to absorb unexpected events (Dück et al., 2012) or to improve the adjustment capabilities (Ionescu & Kliewer, 2011). Considering he/she needs to respect the assumptions and decisions made in the strategic staffing phase, the possible proactive strategies and their magnitude are limited by those assumptions and decisions.

Chapter 3 will further elaborate on the different proactive robustness strategies since this is the topic of this dissertation.

2.3.2 Reactive Personnel Scheduling

Considering it would be very expensive to buffer for every possible unforeseen event in advance by using proactive scheduling strategies, planners usually also make use of reactive scheduling strategies. These involve revising and/or rescheduling when an unexpected event occurs. Some examples of reactive strategies for personnel scheduling will be elaborated on in this section.

Unscheduled overtime

The most commonly used recovery strategy is the use of unscheduled overtime on top of the regular time scheduling. Overtime is used when the staff needed to satisfy the actual demand is higher than the personnel that was scheduled and showed up for his/her shift. Campbell (2012) identified three types of overtime; unscheduled, prescheduled fixed overtime, and prescheduled on-call overtime. The first type, also called holdover time, is decided on when the personnel planner knows with certainty the demand that needs to be met and the supply of workers he/she has at hand for the shift. Overtime can be included by bringing forward the start time of one's shift or by postponing the end time. It is commonly used because it is easy to implement, always available, and can be assigned as soon as a staffing deficit appears (Mac-Vicar et al., 2017).

Since it is so easy to implement, management loves to make use of this strategy. However, employees and the customers they serve have been negatively affected by the use of unscheduled overtime which leads to overall lower productivity. Hospital nurses, for example, suffer from additional stress that can lead to job dissatisfaction and burn-outs and incur an increased error rate on regular tasks, like administrating incorrect medication (Lobo et al., 2013)). The other 2 types of overtime, prescheduled fixed overtime and prescheduled on-call overtime, are examples of proactive strategies and will therefore be explained in Chapter 3 but were included here to be complete.

Conversion of reserve duties

Another strategy to re-establish the balance between staff supply and demand is the conversion of reserve duties. When using reserve duties one faces the trade-off between the wage, cancellation, and change costs for scheduling reserve duties and the cost of shortages when (part of) the actual demand cannot be met. Based on this trade-off Ingels and Maenhout (2015) remarked that the buffer capacity in terms of reserve duties should be a fixed ratio of the minimum staffing requirements. Reserve duties, or employee call-in (Mac-Vicar et al., 2017) refers to employees contracted for a certain minimum guaranteed number of weekly hours but who can be called in for additional shifts. Their wage consists of an hourly wage plus an additional wage for standing by during a certain period when they can be called to start an additional shift. These additional shifts have a minimum length and the worker needs to be notified of it in advance (Mac-Vicar et al., 2017). The social regulations concerning employee call-in can differ in countries or industries.

In the allocation phase, the personnel planner can decide to convert the reserve duties into work duties if demand cannot be met by the scheduled staff. Ingels and Maenhout (2015) proposed two methods to do so. The first, the fixed reactive mechanism, has the constraint that the duties can only be converted to work duties of the same shift. Because this method assumes that the roster cannot be changed in the short term, it offers little flexibility. However, it gives a clear overview of how accurately the reserve duties were scheduled in the original personnel roster.

The reserve duty can be converted to a working duty, can be canceled, or can be changed to reserve duty in another shift in the adjustable reactive mechanism. In the same manner, working duties can be changed to other working duties or can be canceled. Note that this can only be done while all the time-related constraints are still respected. As such, this flexible method comes with an additional trade-off between the number of changes that need to be made and the number of shortages.

Temporary workers

Temporary workers are workers supplied by an external firm who are temporarily hired by a company to assure availability (Mac-Vicar et al., 2017). This reactive strategy has some pros and cons that depend in large measure on the terms of the contract, especially those regarding productivity and availability guarantees (Milner & Pinker, 2001). The cost of training, remuneration, and administration are the main cons of the use of temporary workers.

Reallocation of multi-skilled workers

This strategy entails that workers can be transferred to different departments, which means that they need the different skills required for each of the departments they may be allocated to. This strategy enables shift hours to be dynamically redistributed by making such reallocations for the precise period the contingency lasts without incurring significant labor cost increases (Mac-Vicar et al., 2017). The major cost incurred by this strategy is the training cost and the personnel scheduling cost.

Shift modification and new shift assignment

As indicated by the name, a shift modification is a change made to the existing personnel roster. Generally, shift modifications are only possible if employee acceptance is obtained. Since it does not allow for small local adjustments, this strategy lacks flexibility. Another disadvantage is the fact that workers need to be warned of changes in advance. Again, regulations concerning shift modifications can differ in countries and industries.

New shift assignments involve creating a shift for an employee on a scheduled day off (Mac-Vicar et al., 2017). Most of the time, new shift assignments go hand in hand with the cancellation of a previously assigned shift.

Accept that demand cannot be met

When demand is higher than forecasted and/or when the supply of staff is lower than scheduled, the actual demand cannot be met by the organization. The personnel planner can change this by applying one of the reactive strategies described in this section, or he/she can choose to leave the situation like it is. Meaning that the organizations just accept that demand will not be met due to the mismatch between supply and demand.

All these adjustments mentioned in the previous section take place in the operational allocation phase where, as stated before, the planner is limited in possibilities to make the timetable feasible again. The personnel planner needs to make short-term decisions to respond to the operational variability, but his/her decision freedom is restricted by, i.a., assumptions made in the strategic staffing phase and the tactical scheduling phase. It also needs to be noted that reactive strategies are very costly and may be at the expense of the personnel.

The availability of less expensive options can be facilitated through the use of proactive strategies in the tactical scheduling phase. A combination of both types of strategies, a proactivereactive strategy, leads to a robust personnel roster that ensures flexibility. Accordingly, the flexibility of the schedule needs to be ensured by appropriate proactive scheduling strategies. In this way, the personnel planner has access to more adjustment possibilities, which enable the planner to efficiently restore the feasibility of the timetable with a small number of adjustments and a minimal impact on service level, personnel cost, and personnel satisfaction (Ingels & Maenhout, 2017).



Figure 2: Proactive and Reactive Robustness Strategies

2.4 Robustness and stability in personnel scheduling

Organizations can navigate uncertainty more effectively and efficiently by combining proactive and reactive strategies. A certain degree of built-in robustness is created in the tactical scheduling phase by integrating absorption and adjustment possibilities that can be exploited in the operational allocation phase when a divergence between the assumptions made and the reality occurs. The absorption and adjustment possibilities contribute respectively to the stability and the robustness of the personnel roster.

2.4.1 Stability

Stability describes the grade of the ability of a plan to remain feasible and cost-efficient under variations of the operating environment without major modifications to the plan (Dück et al., 2012). So roster stability reflects the absorption capability of a baseline personnel schedule, i.e. the capability to recover from unexpected events without the intervention of the personnel planner. Ingels and Maenhout (2018) referred to stability as the effectivity of the strategy. The service level and the personnel cost and satisfaction of the baseline schedule can only be ensured if there is a capacity buffer to absorb a demand that is larger than expected. This capacity buffer contains an additional number of skilled personnel scheduled to meet a demand that is larger than forecasted but does not exceed the buffer size. Shader et al. (2001) indicated that stable work schedules result in less work-related stress, lower anticipated turnover, and work satisfaction.

2.4.2 Robustness

Ionescu and Kliewer (2011) and Ingels and Maenhout (2017) both identified a personnel roster to be robust if it is stable and flexible when events occur that lead to a mismatch between the demand and supply of workers. Flexibility is a schedule's ability to adapt to changing environments in operations at a low cost (Ionescu & Kliewer, 2011). In contrast to roster stability that is reached without the intervention of the personnel planner, roster flexibility refers to the degree to which the personnel planner can efficiently and effectively adjust the baseline personnel shift roster to ensure its quality in the operational allocation phase (Ingels & Maenhout, 2015). To ensure the flexibility of the schedule the personnel planner requires adjustment possibilities that have a minimal impact on the schedule and its quality in terms of service level and personnel cost and satisfaction.

Note that mechanisms to improve the stability of the roster have different objectives than those to improve flexibility, but the combination of both can result in a robust schedule. Robust schedules are able to precede uncertainties and have a predefined solution for addressing those uncertainties (Lim & Mobasher, 2011).

2.4.3 Measures

Goren and Sabuncuoglu (2008) developed two surrogate measures for robustness and stability for schedules in a single-machine environment subject to machine breakdowns. They pointed out that when it comes to robustness, the realized performance of the schedule is more important than the expected or planned performance of the initial schedule. That is why the expected realized performance of the system according to certain scheduling criteria is used to measure the robustness of a roster. Examples of these criteria in job shop scheduling are the makespan, total flow time, and tardiness. Applied to personnel scheduling, the criteria can be a minimized total cost (Hazır et al., 2010; Tam et al., 2011).

Robustness measure : E[$f^r(s)$] where $f^r(s)$ depends on the performance of the schedule

When generating robust schedules, one tries to find a schedule *s*^{*} such that:

 $s^* \in \arg \min_{s \in S} \mathbb{E}[f^r(s)]$

On the other hand, when optimizing a schedule in terms of stability, the realized plan should deviate as little as possible from the initial planned schedule. As a result, the sum of absolute deviations is used to measure stability. When generating stable rosters, one tries to find a schedule s^{**} such that:

$s^{**} \in \arg \min_{s \in S} \mathbb{E} \left[\sum_{i \in I} c_i(s) - c_i^r(s) \right]$		
where	$c_i(s)$ represent a particular state in the initial schedule	
where	$c_i^r(s)$ represent a particular state in the realized schedule	
where	the set I represents all the elements included in the performance	
	measure (i.e. tasks, workers,) of the personnel roster	

Examples of the states under investigation can be the job completion times (Mehta and Uzsoy, 1998; Mehta, 1999; O'Donovan et al., 1999) or the job processing times (Wu et al., 1999). When it comes to personnel scheduling, these states can be the propagation of delay (Dück et al., 2012), cancellations of duties, conversions of reserve duties, the number of shortages (Ingels & Maenhout, 2018), etc.

3 Proactive Robustness Strategies

Most literature about the personnel scheduling problem is devoted to meeting demand while pursuing various objectives like employee satisfaction, workplace regulations, etc. But there are not that many papers regarding robustness in personnel scheduling and the different strategies to create robust personnel schedules. Whereas inevitable disruptions compromise the workability of the personnel roster, uncertainty needs to be proactively encapsulated in the original personnel roster by means of robustness strategies that anticipate the realization of unpredicted events.

In this chapter, different strategies will be discussed. Chapters 3.1, 3.2 and 3.3 represent the most commonly used strategies to improve the robustness of personnel rosters. Defining characteristics of uncertainty sets (Chapter 3.4) is used less in practice since it requires probabilistic knowledge of the uncertainty events. In Chapter 3.5 unit crewing is described, a strategy that is only applicable to tasks that can need to be performed in teams.

3.1 Reserve duties

A frequently used strategy to improve the robustness of a schedule is the proactive assignment of reserve duties to the personnel roster. In this way, a demand that is larger than expected can still be met as long as the additional capacity buffer of workers is not exhausted. Ingels and Maenhout (2015) investigated the impact of different strategies to assign those reserve duties to the schedule on the robustness of the personnel roster.

Strategy 1: No reserve duties are introduced in the tactical personnel roster.

Strategy 2: The reserve duties are overstaffed workers on top of the regular working staff requirements.

Strategy3: Reserve duties are introduced based on reserve duty requirements, no reserve time-related constraints are imposed.

Strategy 4: Reserve duties are introduced based solely on reserve time-related constraints.

Strategy 5: A combination of strategies 3 and 4, meaning that reserve duties are introduced based on both reserve duty requirements and reserve time-related constraints.

When they compared the actual performance resulting from strategy 1 to the actual performance of strategy 2 they concluded that the performance of strategy 1 was worse but that none of these two strategies were able to bring the robustness to a satisfactory level. If they considered reserve time-related constraints to compute the reserve duties, the roster has a slightly better ability to handle schedule disruptions than when strategy 2 is used. As strategies 1,2 and 4 result in rather low actual performance, proper scheduling of reserve duties to particular days and shifts is required. This is done by adding the reserve duty staffing requirements in strategies 3 and 5, resulting in a significant decrease in the number of shortages. Reserve duty requirements and reserve time-related constraints should be carefully designed to determine the optimal buffer size and positioning. As such the robustness of the personnel schedule can be improved.

They concluded that capacity buffers, in the form of reserve duties, were necessary to contain the performance of the roster under uncertainty. Of course, extra planned personnel comes at a cost. When deciding on the size of the capacity buffer, there is a trade-off between the extra personnel cost and the cost of staff shortage when the actual demand is higher than expected.

The higher the capacity buffer determined in the scheduling phase, the lower number of shortages but the higher the salary cost and the cost of canceling redundant services when (a part of) the capacity buffer is redundant. The buffer should be sufficient to absorb the unforeseen events but may not be too high to ensure the availability of staff for other tasks.

Considering the personnel number and types are limited by the personnel mix and budget, the optimal additional crew is decided in the strategic staffing phase (Tan, 2003). This first example of a proactive strategy can then be used in the tactical scheduling phase to include time and resource buffers. Time buffers can be used to cope with different types of uncertainties, like unforeseen delays in the personnel task scheduling problem (Tam et al., 2011) for example. Resource buffers can be formed by introducing reserve staff in the tactical scheduling phase or by preferred staffing requirements (Dowsland and Thompson, 2000; Topaloglu and Selim, 2010), these types of requirements are higher than the minimum staffing requirements. In this way, one can absorb uncertainty by incorporating more staff than needed for the expected demand. Including extra staff in the tactical staffing, phase will lead to a higher cost than when no additional staff is incorporated but has the advantage that the increase in cost faced due to the operational variability in the operational allocation phase will be lower as a result of the improved robustness in the personnel roster.

3.2 Overtime

Another example of a proactive strategy is the use of overtime. This combination of time and resources buffers leads to an increase in the resources during a time period and an overall increase in the working time of the employees over all time periods. There are 3 types of overtime; unscheduled overtime, prescheduled fixed overtime, and prescheduled on-call overtime (Campbell, 2012).

The first type, unscheduled overtime, is used in the operational allocation phase, and thus at the moment, uncertainty is faced. Therefore it should be placed with the reactive strategies but was included here to be complete. The two proactive options, prescheduled fixed overtime and prescheduled on-call overtime, each come with their advantages. From a cost perspective, prescheduled on-call overtime has an advantage over fixed overtime since this type is less costly when the worker is not called. But this uncertainty is not beneficial for the worker's well-being and can result in a lower quality of service.

However, the staff's preference in the trade-off between the certainty of an extra shift and the pay for a standby shift when not called differs for each worker. According to Lobo et al. (2013) workers may be more willing to and able to accept overtime if it is offered further ahead of the shift.

The workforce size and overtime have an important impact on the total personnel cost, which contains a significant fraction of the operating costs, of an organization. Both policies, hiring and overtime policy, are intertwined since the amount of hired employees defines the degree to which overtime is required and the ability to include overtime in the personnel roster (Li & Li, 2000). A low number of hired employees results in more overtime that is proactively scheduled in the personnel roster and more cancellations in the operational allocation phase. As such, overtime work may reduce the required number of staff as it improves the flexibility of the personnel roster.

3.3 Employee substitutability

A widely used proactive strategy to improve the flexibility of the personnel roster is the introduction of employee substitutability in the integrated strategic scheduling and tactical scheduling phase. Personnel substitutions happen when a staff member can take over a shift, that requires specific skills, from another working employee on the same day. There are three types of personnel substitutions, namely between-skill, within-skill, and day-off-to-work personnel substitutions. In the former type, substitutions can be made between skills, meaning that a worker can be reassigned from one shift to another shift (on the same day) that requires different skills than the previously assigned shift. Within-skill personnel substitutions can only be done between shifts (on the same day) that require the same set of skills. In day-off-to-work substitutions, an employee who had a day off scheduled is assigned to a working shift based on his/her competencies.

Ingels and Maenhout (2017) observed employee substitutability on two levels, individual employee substitutability and group employee substitutability. The latter requires an additional capacity buffer of skilled workers on top of the minimum staffing requirements during a particular shift. On the level of the individual employee, substitutability depends on the number of substitution possibilities and the value of the substitutions, while substitutability on a group level depends on the degree of cross-training of the assigned workforce. Li and Li (2000) emphasized that the strategic personnel budget should consider the personnel mix and employee cross-training as the availability of cross-trained employees offers a certain leeway to respond to uncertainty.

A higher number of skilled workers can be obtained by scheduling reserve duties (Section 3.1) that have sufficient skills for the tasks or by assigning multi-skilled to other skill duties during the same shift, i.e. employee substitutability (Ingels & Maenhout, 2017). When deciding on the skilled staff size, the personnel planner should bear in mind the trade-off between the extra wage for scheduling additional employees on top of the minimum staffing requirements (reserve duties) and the cost of scheduling skilled workers that are more expensive (employee substitutability). So, reserve duties are used as backups and step in when another worker is absent. Meanwhile, employee substitutability refers to the fact that if all employees possess (almost) the same set of skills, the duties can be interchanged between workers. Employee substitutability offers more flexibility and adaptability within the workforce.

3.4 Defining characteristics of uncertainty sets

This proactive strategy is based on the principles of robust optimization to solve a deterministic formulation of a personnel scheduling problem that is uncertain in terms of personnel demand and employee availability. Robust optimizing is a more recent approach to optimizing under uncertainty, in which the uncertainty model is not stochastic but rather deterministic and setbased (Bertsimas et al., 2011). Gregory et al. (2011) defined robust optimizing as a tractable alternative to stochastic programming particularly suited for problems in which parameters are unknown, variable and their distributions are uncertain. This method tries to minimize the negative impact of future unforeseen events.

Where stochastic optimization assumes that the true probability distribution of uncertainty data has to be known or estimated, robust optimization assumes uncertainty data resides in the so-called 'uncertainty sets' (Gorissen et al., 2015). These uncertainty sets contain values the uncertain personnel demand and employee availability may obtain and are characterized by their structure and scale (Bertsimas et al., 2011).

By incorporating this strategy in the tactical scheduling phase, the personnel planner creates a personnel roster that is robust and feasible for any realization of the uncertainty set and is workable in terms of costs (Gabrel et al., 2014). This approach can also control the risk-taking level of the personnel planner to balance between the optimality and feasibility of the personnel roster (Vitali et al., 2022).

This level of conservatism determines the size of the uncertainty set. Applied to the personnel task scheduling problem, the boundaries of the uncertainty set are determined by the minimum and maximum number of employees that are required to cover the uncertain personnel demand while taking the uncertain employee supply into account. The level of conservatism should be determined by the trade-off between the degree and cost of robustness (Bertsimas & Thiele, 2006), i.e. the cost difference between the robust solution and the minimum cost solution (Gregory et al., 2011). A high degree of conservatism will lead to a very high cost of robustness, therefore it is important to find uncertainty that is appropriate to the problem but is not too conservative. An uncertainty budget, which imposed a maximum on the degree to which parameters may receive a different-than-expected value, was proposed by Bertsimas and Sim (2004) to control the level of conservatism.

3.5 Unit crewing

Unit crewing, defined by Tam et al. (2014) as ensuring that employee teams stay together as long as possible over a sequence of tasks, is another proactive strategy. This strategy was first proposed in the airline industry where they teamed up staff from different ranks and/or crew groups to operate on the same sequence of flights to solve the airline scheduling problem. The possibility of propagation of delay due to its uncertainty is minimized by keeping the crew of different ranks together for as long as possible. Delay is built up when the team needs to wait on another team member to start a certain task because he/she had not yet finished the previous task with another team. This dependency of multiple tasks waiting for employees assigned to an earlier task can be reduced by keeping the teams together as long as possible.

Unit crewing can only be used when tasks need to be completed in teams. Because of this specificity, unit crewing will not be investigated but was stated here to be complete.

3.6 Literature Summary

Table 1 gives an overview of the literature per proactive strategy discussed in Chapter 3.

Literature Summary Proactive Robustness Strategies			
Reserve duties	Overtime		
Dillon and Kontogiorgis (1999) Rosenberger et al. (2002) Abdelghany et al. (2004) Sohoni et al. (2006) Abdelghany et al. (2008) El Moudani and Mora-Camino (2010) Potthoff et al. (2010) Dück et al. (2012) Ingels and Maenhout (2015)	Easton and Rossin (1997) Li and Li (2000) Bertsimas and Sim (2004) Gregory et al. (2011) Campbell (2012) Lobo et al. (2013)		
Employee substitutability	Defining characteristics of uncertainty sets		
Abdelghany et al. (2004) Shebalov and Klabjan (2006) Bailyn et al. (2007) Abdelghany et al. (2008) Eggenberg et al. (2010) Ionescu and Kliewer (2011) Dück et al. (2012) Im et al. (2013) Olivella and Nembhard (2016)	Soyster (1973) Bertsimas and Sim (2004) Shebalov and Klabjan (2006) Gregory et al. (2011)		
Unit Crewing Tam et al. (2014) Shebalov and Klabjan (2006) Ionescu and Kliewer (2011) Tekiner et al. (2009) Saddoune et al. (2012) Weide et al. (2010)			

Table 1: Literature Summary Proactive Robustness Strategies

Part II: Problem description, formulation and methodology
4 Problem description

Most operating systems are managed in a dynamic environment as unpredictable events may disrupt the scheduled plans, making the previously announced schedule infeasible. In the literature, different types of multi-stage decision methodologies have been proposed. With this in mind, solution approaches have been developed relying on stochastic programming or (adjustable) robust optimization to incorporate some of the uncertainty information upfront when constructing the original schedule. Different types of objectives have been provided to restore the workability of the schedule considering robustness measures minimizing the effect of the disruptions on the performance of the realized schedule, and/or stability measures minimizing the deviation from the original schedule. The goal of this dissertation is to study the robustness and stability of proactive scheduling strategies in timetabling applications.

To construct the timetables for the staff, the integrated personnel shift and task scheduling framework of Maenhout and Vanhoucke (2018), described in Chapter 4.1, will be used as the base for all the models in this master dissertation in combination with the research of Cheang et al. (2003). When constructing the minimum-cost baseline personnel roster (Chapter 6), the personnel planner does not yet have any information on the staff supply and demand for that planning period. As a result, assumptions of these parameters need to be made to solve the model. Nevertheless, disruptions that occur due to operational variability may make the original personnel roster infeasible. To anticipate the occurrence of these uncertain events (Chapter 9) one can make use of proactive strategies. The baseline roster (Chapter 6) is expanded to incorporate the proactive strategy (Section 7.1 and 8.1) in the integrated strategic staffing and tactical scheduling phase. Note that the numbers of the equations that were adjusted or added to these models with regard to the baseline model are underlined. In the operational allocation phase, the proactive decisions are implemented on a day-by-day basis stemming from up-to-date information (Section 7.2 and 8.2).

4.1 The integrated Personnel shift and task scheduling problem

An integrated personnel shift and task scheduling baseline roster assigns a set of tasks to a set of available workers, who are organized to work according to shift duties (Maenhout & Vanhoucke, 2018). This framework considers a planning horizon over a set of days D. Each day consists of a set of tasks J_d for that particular day d, where each task $j \in J_d$ has a fixed start \bar{st}_j and finish time \bar{ft}_j and in accordance a fixed duration \bar{t}_j .

An employee is assigned to a shift $s \in S$ on each day $d \in D$, where s^* is considered as a day off. The set of workers W that can be assigned to tasks and shifts is a homogeneous workforce, meaning that every worker w masters the same set of skills. Keep in mind that in this dissertation we make use of a heterogeneous workforce and as such, slight adaptations will be made to the models.

There are two types of constraints imposed by the model, i.e. staffing requirements and timerelated constraints. The staffing requirement R_j is evaluated on an individual level, meaning that a task will be carried out by a worker or not. The time-related constraints contain multiple regulations. For example, a minimum rest time of 11 hours needs to be included in the timetable. Accordingly, a worker can only be assigned to a single shift $s \in S \setminus s^*$ or to a day off on each day d. A minimum and maximum of (consecutive) assignments over the planning horizon is another constraint that needs to be kept in mind. The workforce size and the total number of assignments, making up the wage cost, are minimized in the objective function.

A couple of assumptions made in this model have to be highlighted;

- Lunch breaks are not taken into account.
- There is no pre-emption allowed, meaning that the worker who starts the task has to complete it.

An example of an integrated personnel and task scheduling baseline roster is shown in Figure 3. The planning horizon D contains 4 days, consisting of 3 shifts, and the workforce W contains 7 workers. Each worker is assigned to one of those 3 shifts to cover a set of tasks or to a day off. On day 1 worker 1 is scheduled to shift 1 to perform tasks 1 and 4, while on the next day, day 2, he/she is assigned to a day off.



Figure 3: Integrated Task and Personnel Scheduling Decision (Maenhout & Vanhoucke, 2018)

However, this baseline timetable was constructed based on assumptions about the staffing requirements R_j and the availability of worker w on day d $a_w d$. These assumptions may become inaccurate when more detailed information is available as we get closer to the planning period. The three types of operational variability that may make the personnel roster infeasible (Chapter 2.2) are indicated in Figure 4.

The uncertainty of demand leads to the cancellation of task 8 on day 1 and to the addition of task 10 on day 4. The absence of worker 5 on day 2 is a result of the uncertainty of capacity. Tasks 1 and 2 need to be retimed on day 3 due to uncertainty of arrival, making it impossible for worker 6 to still perform these tasks since they overlap. The feasibility of the roster can be restored by assigning workers 1,3 and 5 to an additional duty on days 2, 3, and 4, respectively. Note that in this dissertation, we will only investigate the uncertainty of demand and capacity.



Figure 4: Occurrence of operational variability (Maenhout and Vanhoucke, 2018)

5 Methodology

The three-step method of Ingels and Maenhout (2015) is used to investigate the impact of proactive strategies on the robustness and stability of the personnel roster. The three steps discussed below, and indicated in Figure 5, will be completed for Chapters 6, 7, and 8. To get a general idea about the robustness and stability of the timetable, step 2 will be iterated 100 times.

- 1. The baseline personnel roster is constructed in the integrated strategical staffing and tactical scheduling phase, without the inclusion of the proactive strategy (Chapter 6) or with inclusion (Chapters 7.1 and 8.1). This MIP is solved in the first step.
- 2. These roster are used as input for the operational allocation phase in the second step. First, a discrete-event simulation is used to simulate the availability and requirements for staff (Chapter 9). Due to diversions from the assumptions made in the original roster, the schedule needs to be adjusted (Chapters 7.2 and 8.2).
- 3. To conclude the adjusted roster is evaluated in terms of robustness and stability The models including reserve duties are also tested against the baseline model without the incorporation of a proactive strategy that was subject to uncertainty but had no options to adjust to it.



Figure 5: Methodology

6 Baseline personnel roster

6.1 The integrated strategic staffing and tactical scheduling phase

In the integrated strategic staffing and tactical scheduling phase, the baseline schedule for a medium period is constructed. In this chapter, the formulation of this roster will be discussed. The construction of the baseline schedule used in this dissertation is based on the integrated personnel shift and task scheduling problem (Chapter 4.1) introduced by Maenhout and Vanhoucke (2018) in combination with the research Cheang et al. (2003). Since their model applies to a homogeneous workforce, slight adaptations had to be made so it would apply to a heterogeneous workforce, meaning that different staff members master a different set of skills.

6.1.1 Model formulation

Notation

Sets

- D The set of days in the planning horizon (index d)
- N The set of weeks in the planning horizon (index n)
- S The set of shifts (index s), including the assignment of having a day off (s*)
- *W* The set of heterogeneous workers (index w)
- J_d The set of tasks (index j) on day d, $\forall d \in D$
- *C* The original set of maximal cliques (index c)
- K_c The original set of tasks incorporated in the maximal clique c
- B_s The set of shifts that cannot be assigned to day + 1 after shift $s \in S$ has been assigned to day d as a result of the minimum rest constraint
- P_i The set of personnel that can perform task j

Parameters

- R_j The original staffing requirements on task $j \in J_d, \forall d \in D$
- g^{max} The maximum number of working assignments in one week for a single worker
- h^{max} The maximum number of consecutive working assignments for a single worker

Decision variables

- x_{jw} 1, if worker $w \in W$ is assigned to task $j \in J_d$, $\forall d \in D$, in the baseline roster roster, 0 otherwise
- y_{wds} 1, if worker $w \in W$ is assigned to shift $s \in S$ on day $d \in D$ in the baseline roster, 0 otherwise
- z_w 1, if worker $w \in W$ is assigned to perform a duty roster in the baseline roster, 0 otherwise

Mathematical formulation

Objective function

Minimize
$$M_1 \sum_{w \in \bar{W}} z_w$$
 (1*a*)

$$+ M_2 \sum_{w \in W} \sum_{d \in D} \sum_{s \in S \setminus s^*} \mathcal{Y}_{wds}$$
^(1b)

$$+ M_4 \sum_{w \in W} \sum_{d \in D} \sum_{s \in S} p_{wds} y_{wds}$$
(1c)

where
$$M_1 > M_2 > M_4$$

Constraints

- Subject to $\sum_{w \in W} x_{jw} = R_j$ $\forall j \in J_d, \forall d \in D$ (2)
 - $\sum_{j \in K_c} x_{jw} \le y_{wd_j s_j} \qquad \qquad \forall w \in W, \forall c \in C \quad (3)$
 - $\sum_{d \in D} \sum_{s \in S \setminus s^*} y_{wds} \le M z_w \qquad \qquad \forall w \in W \quad (4)$
 - $\sum_{s \in S} y_{wds} = 1 \qquad \qquad \forall w \in W, \forall d \in D \quad (5)$
 - $\sum_{d=7n}^{7n+6} \sum_{s \in S \setminus s^*} y_{wds} \le g^{max} \qquad \forall w \in \bar{W}, \forall n \in N \quad (6)$
 - $\sum_{d'=d}^{d+h^{max}} \sum_{s \in S \setminus s^*} y_{wd's} \le h^{max} \qquad \forall w \in W, \forall d \in D \quad (7)$

$\sum_{s\in S\backslash s^*} y_{wds} - \sum_{s\in S\backslash s^*} y_{w(d-1)s} - \sum_{s\in S\backslash s^*} y_{w(d+1)s} \le$	$0 \qquad \forall w \in W, \forall d \in D$	(8)
y_{wds^*} - $y_{w(d-1)s^*}$ - $y_{w(d+1)s^*} \leq 0$	$\forall w \in W, \forall d \in D$	(9)
$y_{wds} + \sum_{s \in B_s} y_{w(d+1)s'} \le 1$	$\forall w \in W, \forall d \in D, \forall s \in S$	(10)
$x_{jw} = 0$	$\forall j \in J_d, \forall d \in D, \forall w \notin \mathbb{P}_j$	(11)
$x_{jw} \leq \sum_{s \in S \setminus s^*} y_{wd_j s_j}$	$\forall w \in W, \forall j \in J_d, \forall d \in D$	(12)
x_{jw} is binary	$\forall j \in J_d, d \in D, \forall w \in W$	(13)
y_{wds} is binary	$\forall w \in W, \forall d \in D, \forall s \in S$	(14)
z_w is binary	$\forall w \in W$	(15)

6.1.2 Model discussion

The objective function of the integrated personnel shift and task scheduling problem minimizes the wage cost, resulting from the workforce size (1a) and the number of assigned shift duties (1b). The former is prioritized as indicated by the coefficient M_1 , which has a larger value than M_2 . As such the number of workers needed to execute all tasks in the planning horizon is minimized. Additionally, personnel satisfaction is respected by minimizing the preference penalty in (1c).

The first constraint (2) ensures that all staffing requirements are met. Constraint (3) assures that a worker cannot be assigned to overlapping tasks. The identification of the set of overlapping tasks K_c will be discussed below. The staffing or hiring constraint (4) entails that a worker is hired if he/she is assigned to a working shift in the planning horizon. Equation (5) assigns a worker to a working shift or to a day off. (6) and (7) constraints the maximum number of (consecutive) working assignments per week. A minimum of 2 consecutive working days or days off is enforced by equations (8) and (9). If a worker is assigned to a working shift on day d, the worker should also be assigned to a working shift on day d-1 and/or on day d+1, the same holds for the assignment of a day off. The minimum rest time of 11 hours between 2 working shifts is respected by constraint (10).

To adapt the integrated personnel shift and task scheduling problem introduced by Maenhout and Vanhoucke (2018) to apply to a heterogeneous (multi-skilled) workforce, constraint (11) was added to the model. This equation implies that a worker cannot be assigned to a task if he/she does not have the required skills for that task. Linking constraint (12) makes sure that a worker is only assigned to tasks that are planned in a shift he/she is assigned to. Equations (13), (14), and (15) make sure that the decision variables are binary.

Identification of the set of overlapping tasks K_c

To ensure that workers cannot be assigned to overlapping tasks, equation (3) is included in the model. This constraint prohibits a worker to be assigned to more than 1 task out of the set of overlapping tasks K_c since otherwise the roster will be infeasible. The polynomial time algorithm of Krishnamoorthy et al. (2012) is used to identify this set of overlapping tasks.

First, the (maximal) cliques need to be identified. A set of tasks $K \subseteq J$ is called *a clique* if all tasks overlap for some interval of time in the planning time horizon. A maximal clique is a clique that cannot be extended further, that is for which no additional task in $J \setminus K$ overlaps with the tasks in the cliques (Krishnamoorthy et al., 2012). To do so, an interval graph G = (J, A) is defined, where J represents the set of nodes and A the set of arcs. In this interval graph, two nodes j,k are connected by an arc if the intervals $[s_j, f_j]$ and $[s_k, f_k]$ overlap with each other. Meaning that the start time of one task will lay before the finish time of the other task, consequently, the tasks are performed at the same time. $C = K_1, ..., K_p$, the set of maximal cliques in this conflict graph, consists of sets $K_t \subseteq J$ such that any two tasks in K_t overlap for some interval of time and K_t is maximal. Accordingly, no task in $J \in K_t$ will overlap with all the tasks in K_t . The polynomial time algorithm of Krishnamoorthy et al. (2012) for identifying all the maximal cliques can be found below.

Algorithm 1. Finding maximal cliques

Initialize: $p = 0, C = \emptyset, s = min_{j \in J}s_j, stop = 0;$ $do \qquad f = min_{j:s \leq f_j}f_j$ p = p + 1 $K_p = j \in J | s_j < f \leq f_j$ $C = C \cup \{K_p\}$ $if \exists s_j \geq f \text{ then } s = min_{s_j \geq f}s_j$ else stop = 1

while

stop = 0

Further, for each $w \in W$, let C^w be the set of maximal cliques in the subgraph of G induced by T_w . T_w is the set of tasks that can be executed by worker w and can be derived from P_j . The elements of C^w will be denoted by K_t^w .

The following example, provided by Krishnamoorthy et al. (2012) illustrates these sets.

Example: Assume a set of tasks with starting and ending times as shown in Figure 6.



Figure 6: Set of tasks (Krishnamoorthy et al., 2012)

The resulting interval graph is shown in Figure 7.



Figure 7: Interval graph (Krishnamoorthy et al., 2012)

Now using the algorithm described above the maximal cliques are defined in the following way:

- $s = s_1, f = f_1$, therefore, $K_1 = \{1, 2, 3\}$
- $s = s_4, f = f_3$, therefore, $K_2 = \{2, 3, 4\}$
- $s = s_5, f = f_4$, therefore, $K_3 = \{4, 5, 6, 8\}$
- $s = s_7$, $f = f_5$, therefore, $K_4 = \{5, 6, 7, 8\}$
- $s = s_9, f = f_9$, therefore, $K_5 = \{8, 9\}$

The cliques are C = {1, 2, 3}, {2, 3, 4}, {4, 5, 6, 8}, {5, 6, 7, 8} and {8, 9}, meaning that one worker cannot perform task 1, 2 and 3 simultaneously. The same holds for tasks 5,6,7, and 8. Assume we have $T_w = \{2, 4, 6, 8\}$ for a particular worker w, who can only execute tasks 2,4,6, and 8. We can then define the set of maximal cliques of this worker $w \in W$ as $C^w = \{\{2, 4\}, \{4, 6, 8\}\}$.

7 Personnel roster with proactively scheduled reserve duties

7.1 The integrated strategic staffing and tactical scheduling phase

In this phase, the personnel budget and roster are determined by incorporating reserve duties proactively into the baseline personnel roster from Chapter 6. This proactive robustness strategy is described in Chapter 3.1. To construct this roster the IPSTP model (Chapter 4.1) is extended by the parameters \bar{R}_{ds}^r and $g^{r,max}$ and by the decision variables n_{ds}^r and \bar{y}_{wds}^r to incorporate reserve duties into the baseline model. How they are included in the model is discussed below. Note that the numbers of the equations that were adjusted or added to the model below with regard to the baseline model are underlined.

7.1.1 Model formulation

Notation

Sets

- D The set of days in the planning horizon (index d)
- N The set of weeks in the planning horizon (index n)
- *W* The set of heterogeneous workers (index w)
- S The set of shifts (index s), including the assignment of having a day off (s*)
- J_d The set of tasks (index j) on day d, $\forall d \in D$
- *C* The original set of maximal cliques (index c)
- *K*_c The original set of tasks incorporated in the maximal clique c
- B_s The set of shifts that cannot be assigned to day + 1 after shift s has been assigned to day d as a result of the minimum rest constraint
- P_j The set of personnel that can perform task $j \in J_d, \forall d \in D$

Parameters

- R_{ds}^r The staffing requirements on shift $s \in S \setminus s^*$ on day $d \in D$ for reserve duties
- R_j The original staffing requirements for task $j \in J_d, \forall d \in D$
- g^{max} The maximum number of working assignments in one week for a single worker

- $g^{r,max}$ The maximum number of reserve duty assignments in one week for a single worker
- h^{max} The maximum number of consecutive working assignments for a single worker
- p_{wds} The shift aversion score for a working or reserve duty during shift $s \in S \setminus s^*$ on day $d \in D$ for worker $w \in W$

Decision variables

- x_{jw} 1, if worker $w \in W$ is assigned to task $j \in J_d$, $\forall d \in D$ in the baseline roster, 0 otherwise
- y_{wds} 1, if worker $w \in W$ is assigned to shift $s \in S$ on day $d \in D$ in the baseline roster, 0 otherwise
- y_{wds}^r 1, if worker $w \in W$ receives a reserve duty during shift $s \in S \setminus s^*$ on day $d \in D$, 0 otherwise
- z_w 1, if worker $w \in W$ is assigned to perform a duty roster in the baseline roster, 0 otherwise
- n_j 1 if task $j \in J_d$, $\forall d \in D$ is understaffed, 0 otherwise
- n_{ds}^r The number of reserve duties short on day $d \in D$ during shift $s \in S \setminus s^*$

Mathematical formulation

Objective function

Minimize
$$M_1 \left(\sum_{w \in W} z_w \right) + M_2 \left(\sum_{w \in W} \sum_{d \in D} \sum_{s \in S \setminus s^*} y_{wds} \right)$$
 (1*a*)

$$+ M_3 \sum_{w \in W} \sum_{d \in D} \sum_{s \in S \setminus s^*} \mathcal{Y}^r_{wds}$$
(1b)

+
$$M_4 \left(\sum_{w \in W} \sum_{d \in D} \sum_{s \in S} p_{wds} y_{wds} + \sum_{w \in \bar{W}} \sum_{d \in D} \sum_{s \in S \setminus s^*} p_{wds} y_{wds}^r \right)$$
 (1c)

$$+ M_5 \sum_{d \in D} \sum_{j \in J_d} n_j$$
 (1d)

$$+ M_6 \sum_{d \in D} \sum_{s \in S \setminus s^*} \mathbf{n}_{ds}^r$$
 (1e)

where
$$M_4 < M_3 \le M_2 < M_1 < M_6 \le M_5$$

Constraints

Subject to	$\sum_{w \in W} x_{jw} + n_j = R_j$	$\forall d \in D, \forall j \in J_d$	<u>(2a)</u>
	$\sum_{w \in W} y_{wds}^r + \mathbf{n}_{ds}^r = R_{ds}^r$	$\forall d \in D, \forall s \in S \smallsetminus s^*$	<u>(2b)</u>
	$\sum_{(j)\in K_c} x_{jw} \leq y_{wds_j}$	$\forall w \in W, \forall c \in C$	(3)
	$\sum_{d\in D}\sum_{s\in S\setminus s^*}y_{wds}\leq Mz_w$	$\forall w \in W$	(4 <i>a</i>)
	$\sum_{d\in D}\sum_{s\in S\setminus s^*}y^r_{wds}\leq Mz_w$	$\forall w \in W$	<u>(4b)</u>
	$\sum_{s\in S} y_{wds} + \sum_{s\in S\backslash s^*} y_{wds}^r = 1$	$\forall w \in W, \forall d \in D$	(5)
	$\sum_{d=7n}^{7n+6} \sum_{s \in S \setminus s^*} y_{wds} \leq g^{max}$	$\forall w \in W, \forall n \in N$	(6 <i>a</i>)
	$\sum_{d=7n}^{7n+6} \sum_{s \in S \setminus s^*} \mathcal{Y}_{wds}^r \leq g^{r,max}$	$\forall w \in W, \forall n \in N$	<u>(6b)</u>
	$\sum_{d'=d}^{d+h^{max}} \sum_{s \in S \setminus s^*} (y_{wd's} + y_{wd's}^r) \le h^{max}$	$\forall w \in W, \forall d \in D$	0 <u>(7)</u>
	$\sum_{s\in S\backslash s^*} y_{wds} - \sum_{s\in S\backslash s^*} y_{w(d-1)s} - \sum_{s\in S\backslash s^*} y_{w(d+1)s} \le 0$	$\forall w \in W, \forall d \in D$	(8)
	\mathcal{Y}_{wds^*} - $\mathcal{Y}_{w(d-1)s^*}$ - $\mathcal{Y}_{w(d+1)s^*} \leq 0$	$\forall w \in W, \forall d \in D$	9 (9)
	$y_{wds} + y_{wds}^r + \sum_{s \in B_s} y_{w(d+1)s'} \le 1$	$\forall w \in W, \forall d \in D, \forall s \in S$	(10)
	$x_{jw} = 0$	$\forall j \in J, \forall w \notin \mathbf{P}_j$	(11)
	$x_{jw} \leq \sum_{s \in S \setminus s^*} \mathcal{Y}_{wd_j s_j}$	$\forall w \in W, \forall j \in J_d, \forall d \in D$	(12)
	x_{jw} is binary	$\forall j \in J_d, \forall d \in D, \forall w \in$	(13)

y_{wds} is binary	$\forall w \in W, \forall d \in D, \forall s \in S (14a)$
y_{wds}^r is binary	$\forall w \in W, \forall d \in D, \forall s \in S (14b)$
z_w is binary	$\forall w \in \bar{W} (15)$
$n_{ds}^r \ge 0$	$\forall d \in D, \forall s \in S \setminus s^* \underline{(16)}$
n_j is binary	$\forall d \in D, \forall j \in J_d \underline{(17)}$

7.1.2 Model discussion

The objective function used to construct a personnel roster with proactively scheduled reserve duties deals with multiple objectives, namely minimizing the wage cost, maximizing personnel satisfaction, and ensuring that the staffing requirements are met. The wage cost, resulting from the workforce size and the number of assigned regular or reserve shift duties, is minimized in (1a) and (1b). Equation (1c) minimizes the personnel preference penalty costs. This preference penalty costs p_{wds} expresses the aversion of worker w towards a particular shift s on day d. Hence, a low value for p_{wds} indicates a higher willingness of worker w towards shift s on day d. Lastly, the staffing requirements are ensured in (1d) by minimizing the understaffing for each shift s and day d. Note that since the wage costs are minimized in (1a), the minimization of overstaffing is also assured. The value of M_5 has to be the largest one since otherwise, the model would simply cancel all the tasks.

Constraint (2a) ensures that the staffing requirements for all tasks $j \in J_d$, $d \in D$, are met. Reserve duties are introduced in the medium-term personnel roster by incorporating specific staffing requirements and/or time-related constraints that consider reserve duties only. In constraint, (2b) the additional staffing requirements for reserve duties are formulated. Constraint (3a) assures that one worker cannot be assigned to overlapping tasks (in the same way as discussed in 6.1.2), both for regular workers. Note that there is no such constraint for reserve workers since they are not yet assigned to tasks but are scheduled as a buffer against uncertainty. Equations (4a) and (4b) provide staffing or hiring constraints.

Additional time-related constraints are introduced for reserve workers, on top of the timerelated constraints for regular workers. Constraint (5) guarantees that a worker can only be assigned to one shift or a day off on day d. The maximum number of (consecutive) working assignments are respected by constraints (6a), (6b), and (7). A minimum of 2 consecutive working days or days off is enforced by constraints (8) and (9), this constraint only holds for regular workers, not for reserve workers. The minimum rest time of 11 hours between 2 assigned shifts for worker w is ensured by constraint (10). Constraints (11) to (16) were added to the model in a similar manner to the formulation of the baseline personnel roster.

7.2 The operational allocation phase

The production planner gets faced with operational variability (Section 2.2), which may make the original personnel roster infeasible. The supply and demand of staff need to be balanced again on a day-by-day basis. The adaptations that can be made are the conversion of reserve duties into working duties or the cancellation of the duty. Working duties can be canceled, but workers cannot be reassigned to another shift since we want to investigate proactive strategies in this dissertation (cf. Dück et al., 2012).

Based on the up-to-date information on the availability a_{wd} of workers and the personnel demand \bar{R}'_{j} , the feasibility is restored at the start of day d. The deterministic model for the operational allocation phase discussed below considers a planning period of one day. The inputs for this model are the original roster from Section 7.1 and the simulated parameters a_{wd} and \bar{R}'_{1} .

7.2.1 Model formulation

Notation

Sets

- *W* The set of heterogeneous workers (index w)
- S The set of shifts (index s), including the assignment of having a day off (s*)
- J'_d The set of tasks (index j) on day d under consideration, adjusted to the operational variability
- *C* The set of maximal cliques (index c)
- K_c The set of tasks incorporated in the maximal clique C'
- B_s The set of shifts that cannot be assigned to day + 1 after shift s has been assigned to day d as a result of the minimum rest constraint
- P'_i The set of personnel that can perform task $j \in J_d, \forall d \in D$

Parameters

- p_{wds} The shift aversion score for a working or reserve duty during shift $s \in S$ on day $d \in D$ for worker $w \in W$
- d The day under consideration in the operational planning phase
- R'_i The simulated staffing requirements for task $j \in J_d$
- a_{wd} 1, if worker $w \in W$ available to work on day d, 0 otherwise

• <i>x</i> _{<i>jw</i>}	1, if worker $w \in W$ is assigned to task $j \in J_d$ in the baseline roster, 0 otherwise
• Y _{wds}	1, if worker $w \in W$ is assigned to shift $s \in S \setminus s^*$ on day d in the baseline roster, 0 otherwise
• y_{wds}^r	1, if worker $w \in W$ received a reserve duty during shift $s \in S \setminus s^*$ on day d in the baseline roster, 0 otherwise

• z_w 1, if worker $w \in W$ is assigned to perform a duty roster in the baseline roster, 0 otherwise

Decision variables

- y'_{wds} 1, if worker $w \in W$ receives a working duty during shift $s \in S \setminus s^*$ on day d , 0 otherwise
- c_{wds} 1, if a regular working duty assigned to worker $w \in W$ for shift $s \in S \setminus s^*$ in the baseline roster is canceled in the adjusted roster, 0 otherwise
- c_{wds}^r 1, if the reserved duty assigned to worker $w \in W$ for shift $s \in S \setminus s^*$ in the baseline roster is canceled in the adjusted roster, 0 otherwise
- x'_{jw} 1, if worker $w \in W$ is assigned to task $j \in J_d$ in the adjusted roster, 0 otherwise
- n'_i the amount of times task $j \in J_d$ is understaffed on day d

Mathematical formulation

Objective function

Minimize
$$M_2 \sum_{w \in W} \sum_{d \in D} \sum_{s \in S \setminus s^*} [y'_{wds} - (y^r_{wds} - c^r_{wds})]$$
 (1a)

$$+ M_3 \sum_{w \in W} \sum_{d \in D} \sum_{s \in S \setminus s^*} [y'_{wds} - (y_{wds} - c_{wds})]$$

$$\tag{1b}$$

+
$$M_4 \sum_{w \in \bar{W}} \sum_{s \in S \setminus s^*} p_{wds} y'_{wds}$$
 (1c)

$$+ M_5 \sum_{j \in J_d} n'_j \tag{1d}$$

$$+ M_9 \sum_{w \in W} c_{wds} + M_{10} \sum_{w \in W} c_{wds}^r$$
(1e)

where $M_4 < M_{10} < M_9 \le M_2 \le M_3 < M_5$

Constraints

Subject to	$\sum_{w \in W} x'_{jw} + n'_j = R'_j$	$\forall j \in J_d (2)$
	$\sum_{s\in S\smallsetminus s^*} y'_{wds} \leq a_{wd}$	$\forall w \in W, \forall s \in S \setminus s^* (3)$
	$y_{wds} = y_{wds}' + c_{wds}$	$\forall w \in W, \forall s \in S \setminus s^* (4a)$
	$y_{wds}^r = y_{wds}' + c_{wds}^r$	$\forall w \in W, \forall s \in S \setminus s^* (4b)$
	$\sum_{(j)\in K_c} x'_{jw} \leq y'_{wds_j}$	$\forall w \in W, \forall c \in C (5)$
	$x'_{jw} \leq \sum_{s \in S \setminus s^*} \mathcal{Y}'_{wd_j s_j}$	$\forall w \in W, \forall j \in J_d, \forall d \in D (6)$
	\bar{x}'_{jw} is binary	$\forall w \in \bar{W}, \forall J \in J_d (7)$
	y'_{wds} is binary	$\forall w \in W, \forall s \in S (8)$
	c_{wds} is binary	$\forall w \in W, \forall s \in S (9a)$
	c_{wds}^r is binary	$\forall w \in W, \forall s \in S (9b)$
	$n_j' \geq 0$	$\forall w \in W, \forall s \in S (10a)$
	n'_j is integer	$\forall w \in W, \forall s \in S (10b)$

7.2.2 Model discussion

The objective function stays (almost) the same as in Section 7.1, however, the total cost needs to be adapted. When a duty, assigned in the original roster, gets canceled, a duty cancellation cost needs to be added to the total assignment cost. The worker gets a day off if his/her originally scheduled gets canceled. We set the cost M_10 at a lower value than M_9 in other that reserve duties will get canceled before regular duties get canceled.

Note that in this dissertation we chose to use the fixed reactive mechanism in which no reallocations in terms of shifts are allowed, except for the conversion of reserve duties. In this way, we can evaluate the robustness and stability resulting from proactive strategies. This method is completely dependent on the roster made up in the integrated strategical staffing and tactical scheduling phase (Section 7.1) resulting in less flexibility than an adjustable reactive mechanism.

The operational staffing requirements constraint (2) specifies that the model tries to assign all tasks to regular or duty workers. This demand for staff R'_j is the staffing requirement R_j from Section 7.1 adapted to the operational variability. A staffing requirement that is not met will be denoted by n'_j , which is minimized in the objective function (1c). Note that n'_{ds} is not minimized in the objective function anymore since there are no specific staffing requirements for reserve duties anymore.

The actual availability a_{wd} of worker w on day d is taken into account in constraint (3). This simulated parameter has a value of 0 when worker $w \in W$ is not available on day $d \in W$, and 1 if he/she is available to work on day d. Equation (4a) ensures that a worker is scheduled to a working duty or to a day off when his working duty gets canceled (c_{wds} or c_{wds}^r is equal to 1).

Every reserve duty is converted to a working duty or to a day off when the reserve duty is canceled (4b). This equation also takes the availability a_{wd} for this worker into consideration, if he/she is not available on that d, the duty gets canceled. Constraints (5) and (6) are the same as in the model from Section 7.1, considering that a worker cannot be assigned to overlapping tasks still hold in the operational allocation phase. Constraints (7) to (10b) are the integrality constraints.

8 Personnel roster with proactively scheduled overtime

8.1 The integrated strategic staffing and tactical scheduling phase

Just as in the previous chapter, the personnel budget and roster are determined by incorporating scheduled overtime proactively into the baseline personnel roster from Chapter 6. This proactive robustness strategy is described in Chapter 3.2. To construct this roster, the IPSTP model (Chapter 4.1) is extended by the parameters \bar{R}_{ds}^o and l_{wd}^{max} and by the decision variables \bar{f}_{wd} , \bar{s}_{wd} and n_j to incorporate overtime into the baseline model. How they are included in the model is discussed below. Note that the numbers of the equations that were adjusted or added to the model below with regard to the baseline model are underlined.

8.1.1 Model formulation

Notation

Sets

- D The set of days in the planning horizon (index d)
- N The set of weeks in the planning horizon (index n)
- *W* The set of heterogeneous workers (index w)
- S The set of shifts (index s), including the assignment of having a day off (s^*)
- J_d The set of tasks (index j) on day $d \in D$
- *C* The original set of maximal cliques (index c)
- *K*_c The original set of tasks incorporated in the maximal clique c
- B_s The set of shifts that cannot be assigned to day + 1 after shift $s \in S$ has been assigned to day d as a result of the minimum rest constraint
- P_j The set of personnel that can perform task $j \in J_d, \forall d \in D$

Parameters

- R_j The original staffing requirements on task $j \in J_d, \forall d \in D$
- R_{ds}^{o} The additional staffing requirements on shift $s \in S \setminus s^{*}$ on day $d \in D$, expressed in minutes
- S_s The start time of shift $s \in S \setminus s^*$

- F_s The finish time of shift $s \in S \setminus s^*$
- g^{max} The maximum number of working assignments in one week for a single worker
- h^{max} The maximum number of consecutive working assignments for a single worker
- l_{wd}^{max} The maximum number of regular and overtime hours that can be assigned to worker $w \in W$ on day $d \in D$
- p_{wds} The shift aversion score for a working or reserve duty during shift $s \in S$ on day $d \in D$ for worker $w \in W$

Decision variables

• <i>x</i> _{jw}	1, if worker $w \in W$ is assigned to task $j \in J_d$, $\forall d \in D$ in the baseline roster, 0 otherwise
• Ywds	1, if worker $w \in W$ is assigned to shift $s \in S$ on day $d \in D$ in the baseline roster, 0 otherwise
• f _{wd}	the latest finish time of worker $w \in W$ on day $d \in D$
• S _{wd}	the earliest start time of worker $w \in W$ on day $d \in D$
• Z _w	1, if worker $w \in W$ is assigned to perform a duty roster in the baseline roster, 0 otherwise
• <i>n</i> _j	1 if task <i>j</i> ∈ J_d , $\forall d \in D$ is understaffed, 0 otherwise

• n_{ds}^0 The minutes overtime short on day $d \in D$ during shift $s \in S \setminus s^*$

Mathematical formulation

Objective function

Minimize
$$M_1 \left(\sum_{w \in W} z_w \right) + M_2 \left(\sum_{w \in W} \sum_{d \in D} \sum_{s \in S \setminus s^*} y_{wds} \right)$$
 (1*a*)

$$+ M_7 \left(\sum_{w \in W} \sum_{d \in D} \left[\left(\sum_{s \in S \setminus s^*} y_{wds} S_s - s_{wd} \right) + \left(f_{wd} - \sum_{s \in S \setminus s^*} y_{wds} F_s \right) \right] \right)$$
(1b)

$$+ M_5 \sum_{j \in J} n_j$$
 (1c)

$$+ M_8 \sum_{d \in D} \sum_{s \in S \setminus s^*} n_{ds}^o$$
(1d)

$$+ M_4 \left(\sum_{w \in \bar{W}} \sum_{d \in D} \sum_{s \in S} p_{wds} y_{wds} \right)$$
(1e)

where
$$M_7 \leq M_2 \leq M_1 \leq M_5$$

Constraints

Subject to	$\sum_{w \in W} x_{jw} + n_j = R_j$	$\forall d \in D, \forall j \in J_d (2a)$
	$\sum_{w \in W} \left[(y_{wds} S_s - s_{wd}) + (f_{wd} - y_{wds} F_s) \right] + r$	$a_{ds} = R^o_{ds} \forall d \in D, \forall s \in S \setminus s^* (2b)$
	$\sum_{(j)\in K_c} x_{jw} \leq y_{wd_js_j}$	$\forall w \in W, \forall c \in C (3)$
	$\sum_{d\in D}\sum_{s\in S\smallsetminus s^*}y_{wds}\leq Mz_w$	$\forall w \in W (4)$
	$\sum_{s\in S} y_{wds} = 1$	$\forall w \in W, \forall d \in D (5)$
	$\sum_{d'=d}^{d+h^{max}} \sum_{s \in S \setminus s^*} y_{wd's} \leq h^{max}$	$\forall w \in W, \forall d \in D (6)$
	$\sum_{d=7n}^{7n+6} \sum_{s \in S \setminus s^*} y_{wds} \leq g^{max}$	$\forall w \in \bar{W}, \forall n \in N (7)$
	$\sum_{s\in S\smallsetminus s^*} y_{wds} - \sum_{s\in S\smallsetminus s^*} y_{w(d-1)s} - \sum_{s\in S\smallsetminus s^*} y_{w(d+1)s} - \sum_{s\in S\setminus s^*} y_{w(d+1)s} - \sum_{s$	$\forall w \in W, \forall d \in D$ (8)
	y_{wds^*} - $y_{w(d-1)s^*}$ - $y_{w(d+1)s^*} \leq 0$	$\forall w \in \bar{W}, \forall d \in D (9)$
	$s_{wd} \leq y_{wds}S_s$	$\forall w \in W, \forall d \in D, \forall s \in S \setminus s^* (10a)$
	$f_{wd} \ge y_{wds} F_s$	$\forall w \in W, \forall d \in D, \forall s \in S \setminus s^* (10b)$
	$s_{w(d+1)}$ + 1440 - $f_{wd} \ge 660$	$\forall w \in W, \forall d \in D \setminus \{ D - 1\} (10c)$
	f_{wd} - $s_{wd} \leq l_{wd}^{max}$	$\forall w \in W, \forall d \in D (11)$

$$x_{jw} = 0$$
 $\forall j \in J, \forall w \notin P_j$ (12) $x'_{jw} \leq \sum_{s \in S \setminus s^*} y'_{wd_j s_j}$ $\forall w \in W, \forall j \in J_d, \forall d \in D$ (13) x_{jw} is binary $\forall j \in J_d, \forall d \in D, \forall w \in W$ (14) y_{wds} is binary $\forall w \in W, \forall d \in D, \forall s \in S$ (15) z_w is binary $\forall w \in W, \forall d \in D, \forall s \in S$ (15) n_j is binary $\forall y \in J_d, \forall d \in D$ (17) $n'_{ds} \geq 0$ $\forall d \in D, \forall s \in S \setminus s^*$ (18a) n'_{ds} is integer $\forall d \in D, \forall s \in S \setminus s^*$ (18b)

8.1.2 Model discussion

The objective function (1) considers multiple objectives, i.e. minimizing the wage cost for regular working duties (1a), minimizing the wage cost for overtime (1b), and minimizing the number of understaffed tasks (1c) and minutes of overtime (1d) while maximizing personnel satisfaction (1e). The overtime is calculated per minute for each worker $w \in W$ and each day $d \in D$. The weight of each objective is indicated by a big coefficient M, where $M_1 \leq M_2 \leq M_3$, M_3 has to be the biggest weight because otherwise, the model would just cancel all the tasks.

Staffing requirements are ensured in constraint (2) in the same manner as in the baseline personnel roster (Chapter 6). Constraint (3) makes sure that workers cannot be assigned to overlapping tasks. The staffing or hiring constraint (4) and the other time-related constraints, equations (5) to (9) stay the same as in the baseline roster in Chapter 6.

Equations (10a) and (10b) were added to the model to make sure that overtime can only be assigned to a worker that is assigned to a working shift on that d, not to a worker that has a day off. The minimum rest time of 11 hours, or 660 minutes, is respected by constraint (10c). The number of regular and overtime hours a worker can be assigned per day are restrained to a max of l_{wd}^{max} in constraint (11). Equations (14) to (17) make sure that all decision variables are binary. n_{ds}^r can be any integer bigger than 0 (18a) and (18b).

8.2 The operational allocation phase

Each day $d \in D$ is revised separately in the operational allocation phase since the operational variability (Chapter 2.2) occurs on the day of the operation. Consequently, this model will need to be solved for each day in the planning horizon D. To incorporate the operational variability, overtime was proactive scheduled in the baseline model in the previous section, Section 8.1.

In the operational phase, the personnel planner can cancel the working duties (regular and overtime), but duties cannot be changed since we make use of the fixed reactive mechanism to exclude the impact of reactive actions on the robustness and stability of the personnel roster. This involves changes in shift assignments, tasks assignments however can be swapped.

8.2.1 Model formulation

Notation

Sets

- *W* The set of heterogeneous workers (index w)
- S The set of shifts (index s), including the assignment of having a day off (s*)
- J_d The set of tasks (index j) on day d
- *C* The original set of maximal cliques (index c)
- *K*_c The original set of tasks incorporated in the maximal clique c
- B_s The set of shifts that cannot be assigned to day + 1 after shift s has been assigned to day d as a result of the minimum rest constraint
- P_j The set of personnel that can perform task j

Parameters

- d The day under consideration in the operational planning phase
- R'_i The simulated staffing requirements on task $j \in J_d$
- S_s The start time of shift $s \in S \setminus s^*$
- F_s The finish time of shift $s \in S \setminus s^*$
- x_{jw} 1, if worker $w \in W$ is assigned to task $j \in J_d$ in the baseline roster, 0 otherwise

- y_{wds} 1, if worker $w \in W$ is assigned to shift $s \in S \setminus s^*$ on day d in the baseline roster, 0 otherwise
- f_{wd} the latest finish time of worker $w \in W$ on day d
- s_{wd} the earliest start time of worker $w \in W$ on day d
- z_w 1, if worker $w \in W$ is assigned to perform a duty roster in the baseline roster, 0 otherwise
- n_j 1 if task $j \in J_d$ is understaffed, 0 otherwise
- p_{wds} The shift aversion score for a working or reserve duty during shift $s \in S$ on day d for worker $w \in W$

Decision variables

- x'_{jw} 1, if worker $w \in W$ is assigned to task $j \in J_d$ in the adjusted roster, 0 otherwise
- y'_{wds} 1, of worker $w \in W$ receives a working duty during shift $s \in S \setminus s^*$ on day d under consideration, 0 otherwise
- n'_i The number of understaffing for task $j \in J_d$
- c_{wds} 1, if the duty assigned for shift $s \in S \setminus s^*$ to worker w $w \in W$ in the baseline roster is cancelled in the adjusted roster, 0 otherwise
- f'_{wd} the latest finish time of worker $w \in W$ on day d in the adapted roster
- s'_{wd} the earliest start time of worker $w \in W$ on day d in the adapated roster

Mathematical formulation

Objective function

Minimize
$$M_2 \sum_{w \in W} \sum_{s \in S \setminus s^*} y'_{wds}$$
 (1*a*)

$$+ M_4 \sum_{w \in W} \sum_{s \in S \setminus s^*} p_{wds} y'_{wds}$$
^(1b)

 $+ M_5 \sum_{j \in J_d} n'_j \tag{1c}$

$$+ M_9 \sum_{w \in W} c_{wds} \tag{1d}$$

$$+ M_7 \sum_{w \in W} f'_{wd} + s'_{wd} \tag{1e}$$

where $M_7 < M_2 \le M_4 \le M_9 < M_5$

Constraints

Subject to	$\sum_{w \in W} x'_{jw} + n'_j = R'_j$	$\forall s \in S \setminus s^* (2)$
	$\sum_{s\in S\smallsetminus s^*} y'_{wds} \leq a_{wd}$	$\forall w \in W (3)$
	$y_{wds} = y'_{wds} + c_{wds}$	$\forall w \in W, \forall s \in S \setminus s^* (4)$
	$S_s y_{wds} \ge s'_{wd} \ge s_{wd}$	$\forall w \in W, \forall d \in D$, $\forall s \in S \setminus s^*(5a)$
	$\mathbf{f}_{wd} \geq f_{wd}' \geq F_s y_{wds}$	$\forall w \in W, \forall d \in D, \forall s \in S \setminus s^* (5b)$
	x'_{jw} is binary	$\forall w \in W, \forall J \in J_d (6)$
	y'_{wds} is binary	$\forall w \in W, \forall s \in S (7)$
	c_{wds} is binary	$\forall w \in W, \forall s \in S (8)$
	n'_j is binary	$\forall w \in W, \forall s \in S (9)$

8.2.2 Model discussion

Equivalently as in Chapter 7.2, the total cost needs to be adapted in the operational allocation phase. The total cost now includes the wage cost for regular and overtime, the cancellation cost, and the cost for understaffing.

The staffing requirements from the integrated strategic staffing and tactical scheduling phase in Section 8.1) are adapted to the operational variability (2). The staffing requirements R'_j for task $j \in J_d$ are simulated and used as input for the model in the operational allocation phase. These requirements will be simulated in the computational experiments where the operational variability is simulated, meaning that the value of R'_j can go from a value of 1 to a value of 0 due to uncertainty of demand for example. This uncertainty can arise when the actual demand is lower than expected. The opposite is also possible, a requirement can be augmented by 1. The variable n'_j will be augmented by 1 each time a staffing requirement is not met. This underallocation of tasks is minimized in the objective function by assigning the value M_5 the largest weight.

The same equations hold to enforce that workers cannot be assigned to overlapping tasks as in Section 8.1, but constraint (3) was adapted to take the (simulated) availability of workers into account. A worker can be absent on day d due to illness for example. In that case a_{wd} will receive the value of 0.

Constraint (4) makes sure that a working assignment is either canceled or carried out. The actual start time of worker w needs to lay between the planned earliest start time and the start time of the shift he/she is assigned to. The same reasoning holds for the actual finish time, which needs to lay between the finish time of the shift he/she is assigned to and the planned latest finish time. This is enforced by constraints (5a) and (5b). The other constraints, (6) to (9), have not changed with regard to the baseline model including proactively scheduled overtime. They are just applied to the decision variables of the operational allocation model.

9 The occurrence of operational variability

The models constructed in the tactical phase are predicated on certain assumptions regarding the demand and supply of staff. However, it is important to acknowledge that these assumptions may deviate from reality due to the inherent variability encountered in operational settings. Consequently, the accuracy of these assumptions may be compromised, thereby affecting the feasibility of the personnel roster.

To address this limitation and account for the uncertain events that can influence the staffing requirements (R'_j) and staff availability (a_{wd}) , a simulation approach will be employed. This chapter aims to provide a comprehensive understanding of how these critical parameters will be simulated, thereby enabling a more realistic representation of the dynamic nature of personnel scheduling. Through the simulation process, a range of possible scenarios will be generated, reflecting the diverse factors that can impact staffing needs and staff availability. By incorporating this variability, the resulting personnel roster will be more robust and adaptable, better aligned with the dynamic operational environment.

9.1 The Operational Staffing Requirements

Chapter 2.2 stated that the unpredictability of workload is the result of the uncertainty of demand. This uncertainty impacts the number of tasks that need to be performed each day J_d and as such, the number of staff required. Tasks can be removed due to this unpredictability, but tasks can also be added to the set of tasks for each day. When the actual demand faced in the operational phase is larger than expected, the staffing requirements R'_j will be augmented by one. In other words, a random task of that shift will be duplicated.

Since the Poisson distribution is generally employed for demand for services (Ahmed and Alkhamis, 2009; Yeh and Lin, 2007), this distribution is implemented to simulate operational staffing requirements. The number of tasks in a shift is used as the mean λ . Figure 8 shows the Poisson distribution for the early shift on day 1 with a mean value of 20. A Poisson distribution is slightly skewed to the right, resulting in a greater probability of obtaining a value greater than the mean λ . But this only holds for small values of λ , given the fact that we have a rather large number of assignements per shift, the distribution will be a bell-shaped, symmetric one. This λ represents not only the expected value but also the variance of the distribution. The larger λ is, the larger is also the variance of the distribution. R'_j is generated for each day separately, meaning that the demand over the different days in the planning horizon is independent.



Figure 8: Poisson Distribution for the Number of Tasks in the Early Shift on Day 1

9.2 Employee availability

Illness, for example, can lead to the unavailability of a worker. This can result in a lack of capacity because this worker is no longer capable to perform the task(s) he/she was assigned to. The employee availability parameter a_{wd} can receive a value of zero or one, as such it follows the Bernoulli distribution that is determined by the probability of unavailability $P_{wd}(X = 0)$ (Ingels & Maenhout, 2015). They calculated this probability based on the following formula, i.e.

$$P_{wd}(X = 0) = P(X=0) * f(\text{days absent}_{wd}), \quad \forall w \in W, \forall d \in D$$

The general probability P(X = 0) that a worker is unavailable is multiplied with a decreasing function $f(days \ absent_{wd}) = q^{days \ absent_{wd}}$, where q is a constant value, that has a maximum value of one. In this way, the probability that a worker will be absent decreases as the number of days absent increases, making the absence of workers dependent events in the simulation (Barmby et al., 2002).

According to a study on the sickness absence percentage in 2017 performed by SD Worx (2013), the probability P(X = 0) is equal to 2.44 %. Ingels and Maenhout (2015) determined the value of q to be 0.8158 based on empirical experimentation such that P(X = 0) approximates 0 after 28 days of absence. Therefore, the formula below will be used to determine a_{wd} for each worker on each day.

$$P_{wd}(X=0) = 2.44\% * 0.8158^{days \ absent_{wd}}, \quad \forall w \in W, \forall d \in D$$

Part III: Computational experiments and results

10 Test design

This chapter presents computational insights into the methodology employed throughout this dissertation. In Section 10.1, the test design offers a comprehensive examination of the specific parameter configurations employed during the integrated strategical and tactical staffing phase. Furthermore, Section 10.2 provides a comprehensive account of the parameter settings utilized during the operational allocation phase, encompassing both the simulation and adjustment steps. Section 11.2 describes the framework that will be used in the evaluation of the personnel rosters.

10.1 The integrated strategical staffing and tactical scheduling phase

The data sets generated by Krishnamoorthy et al. (2012) were utilized. They generated 137 data files to evaluate and compare the performance of algorithms for solving the personnel task scheduling problem. Since the integrated personnel task and shift scheduling problem is the scope of this dissertation, some slight adaptions were made to the data sets. In order to get a scheduling horizon of 7 days, 7 different data files were pasted together and their tasks were randomly distributed to the 7 days of the scheduling horizon. If tasks were scheduled over multiple shift they were split up in 2 different tasks such that there are no tasks that overlap shifts.

10.1.1 Personnel characteristics

A heterogeneous workforce of 150 employees can be assigned to a weekly shift pattern. However, not all the workers need to be used since we minimize the workforce size. Each staff member masters a different set of skills resulting in a particular set of tasks he/she can perform. These skills were generated by a multi-skilling level of 33% or 66%. Note that the reserve workers are also part of the set of workers W, thus all workers, regular and reserve, have the same type of personnel characteristics.

The preferences are additionally considered when rostering staff. Preference aversion scores p_{wds} are calculated by randomly assigning a value of 1 to 4 for each combination of worker w, shift s, and day d. No duplicates are allowed. An aversion score of 1 indicates a high preference to be assigned to that shift, on the contrary, a high aversion results in a score of 4. Table 2 shows the shift aversion scores for worker 1 for each shift s and day d, p_{1ds} . The table indicates that worker 1 has the highest preference to work a late shift on day 1, and the lowest preference to work a night shift on day 1.

The shift aversion scores for worker 1				
	Early Shift	Late Shift	Night Shift	Day Off
Day 1	2	1	4	3
Day 2	4	2	1	3
Day 3	2	1	4	3
Day 4	3	1	2	4
Day 5	1	3	2	4
Day 6	1	4	3	2
Day 7	2	3	4	1

Table 2: The shift aversion scores for worker 1

10.1.2 Shift characteristics

The set of shifts S includes 4 shifts: an early, a late, a night shift, and a day off. Each working shift has a duration of 8 hours and there is no overlap between shifts. The shift's start and finish times can be found in Table 3. Note that lunch breaks are not taken into account when we schedule shifts and tasks in this dissertation.

The set of shifts S		
Early shift	6.00 am - 14.00 pm	
Late shift	14.00 pm - 22.00 pm	
Night shift	22.00 pm - 6.00 am	
Day off	0.00 am - 0.00 pm	

Table 3: The set of shifts S

10.1.3 Task characteristics

The number of tasks per day in the scheduling horizon is determined by randomly distributing the tasks from 7 different data files of Krishnamoorthy et al. (2012) over the scheduling horizon. On average 35 tasks need to be executed per shift. Table 4 shows the number of tasks per shift per day, which make a total of 734 tasks to be performed over the 7 days. The requirements for the tasks in this dissertation are always equal to one, meaning that only one worker is assigned to the job. A task is not executed by a team of workers. Furthermore, preemption is not allowed, i.e., the worker who started the job must finish it.

An overview of the artificial set of data for the early shift is shown in Figure 9. In the early shift on day 1, 20 tasks need to be performed. Note that the time axis starts at 0 minutes, meaning 6.00 am, and ends at 480 minutes, meaning 14.00 pm.

The number of tasks per shift per day				
	Early Late Shift Night Total			
	Shift		Shift	
Day 1	20	48	36	104
Day 2	33	30	27	90
Day 3	29	37	30	96
Day 4	35	44	41	120
Day 5	34	54	31	119
Day 6	26	38	26	90
Day 7	36	50	29	115

Table 4: The number of tasks per shift per day

To incorporate reserve duties and overtime proactively, additional staffing requirements are imposed on top of R_j . The model in Section 7.1 introduces additional staffing requirements R_{ds}^r on shift s on day d for reserve duties. The personnel roster with proactively scheduled overtime (Section 8.1) incorporates additional staffing requirements R_{ds}^o on shift s on day d, expressed in minutes.

In accordance with Ingels and Maenhout (2015), the additional requirements R_{ds}^r are determined based on the total staff size for that particular shift s and day d and a buffer size ratio b (Req. 1). The additional requirements for overtime are assessed in the same way but the buffer size indicates how many workers will be assigned to an equal amount of overtime. As such, to get the total additional requirements, the buffer size needs to be multiplied by the minutes of overtime the workers will be assigned (Req 2.). By assigning employees the same amount, the required prescheduled overtime is fairly distributed over the staff.

The numbers are rounded to the nearest integer since we don't allow the requirements to be fractional numbers. Multiple values for b will be used in the simulation.

$$R_{ds}^{r} = \text{ROUND} \left[\sum_{w \in W} y_{wds} * b \right]$$
 (Req. 1) $\forall d \in D, s \in S \setminus s^{*}$

$$R_{ds}^{o} = \text{ROUND} \left[\sum_{w \in W} y_{wds} * b \right] * \text{minutes}_{overtime} \qquad (\text{Req. 2}) \quad \forall d \in D, s \in S \setminus s^{*}$$



TIME (IN MINUTES)

Figure 9: Tasks Early Shift on Day 1

10.1.4 Time-related constraints

Several time-related constraints are imposed in the models discussed in Chapters 6, 11.1.2, and 11.1.3. The models make sure that a worker is always assigned to a working shift or to a day off. Another constraint enforces a maximum number of (consecutive) assignments. In addition, a minimum rest period of 11 hours between consecutive shifts is imposed by making use of the set B_s . The values for the parameters and the set B_s are indicated in the table below.

	Time-related constraint parameters			
g ^{max}	5	A worker can be assigned to at most 5 shift assignments per week.		
h ^{max}	5	A worker can be assigned to at most 5 consecutive shift assignments.		
B _s	Early Shift	An early shift can be followed by any type of shift.		
	Late Shift	A late shift can be followed by a late shift, night shift, and a day off.		
	Night Shift	A night shift can only be followed by a night shift or a day off.		
	Day off	A day off can be followed by any type of shift.		

Table 5: Time-related constraint parameters

10.1.5 Objective Function Parameters

The models used in the integrated strategic staffing and tactical scheduling phase minimize the total cost incurred by the personnel roster. This total cost is composed of several parts. The weights given to each part are described in Table 6. The cost of understaffing was set rather high because we prioritize meeting the customer's demand at each time. Note that the value of big M used in constraints (4) and (5) in Chapters 6, 7.1, and 8.1 gets a value of 1.000. The values for each parameter are in accordance with Ingels and Maenhout (2015) and Ingels and Maenhout (2018).

Objective Function Parameters - Tactical Staffing Phase		
<i>M</i> ₁	40	The workforce size
M_2	30	The wage cost of regular workers
M_3	20	The wage cost of reserve workers
M_4	2	The preference penalty cost
M_5	85	The cost of understaffing a certain task
M_6	80	The cost of understaffing reserve duties during a certain shift
M_7	0.09375	The wage cost per minute of overtime
M_8	0.125	The cost of understaffing a minute of overtime

Table 6: Objective Function Parameters - Tactical Staffing Phase

10.2 The operational allocation phase

Referring to Chapter 5, the operational allocation phase is formed by two sequential steps, the simulation step, and the adjustment step. Both steps were repeated 100 times to get general results to evaluate the stability and robustness created by proactively incorporating reserve duties and/or overtime. The two steps and how they will be evaluated are described in the following sections.

10.2.1 Simulation step

In this first stage of the operational allocation phase, the staffing requirements and the availability of employees are simulated to duplicate the uncertainty of the stochastic operational environment organizations operate in. Chapter 9 described how these parameters are simulated. When the requirements in the operational phase R'_j differ from the ones in the tactical scheduling phase R_j , the tasks are either duplicated or deleted from the set of tasks J_d with the use of a random number generator that selects the tasks that will be duplicated or deleted.

10.2.2 Adjustment step

The next undertaking is the adjustment step, where the baseline roster with the incorporation of the proactive strategy is adapted to the up-to-date information on a daily basis. The updated information about the staffing requirements and employee availability is the result of the previous step.

To solely study the impact of the proactive incorporation of strategies we chose to prohibit swaps in the allocation phase. As such, a duty scheduled in the integrated staffing and scheduling phase can either be performed or canceled. Constraints (4a) and (4b) in Section 7.2 and 8.2 enforce this. The objective function parameters used in the operational allocation phase are presented in Table 7. The cost of canceling a regular duty is set higher than the one that occurred when a reserve duty is canceled since we want to cancel reserve ones first.
	Objective Fu	nction Parameters - Operational Allocation Phase
M_1	40	The workforce size
M_2	30	The wage cost of regular workers
M_3	35	The wage cost of reserve workers
M_4	5	The preference penalty cost
M_5	85	The cost of understaffing a certain task
M_9	25	The cancellation cost occurred when a regular duty is cancelled
M ₁₀	15	The cancellation cost occurred when a reserve duty is cancelled

Table 7: Objective Function Parameters - Operational Allocation Phase

10.3 Evaluation

To evaluate the impact of the inclusion of reserve duties and/or overtime in the baseline model in terms of stability and robustness, the following evaluation framework will be used.

Stability

Roster stability reflects the absorption capability of a personnel roster, i.e. the ability of a roster to remain feasible and workable in a stochastic environment without major modifications to the originally planned timetable. In light of this, the different deviations will be examined. The different measures are provided in Table 8. In accordance with Dück et al. (2012), we chose to not allow swaps, since we only want to determine the impact of proactive strategies.

A personnel roster can be adapted in the operational allocation phase by canceling regular and reserve workers and by conversing reserve duties to working duties. Task understaffing, leading to task cancellations, are deviations to the output of the personnel roster and will therefore also be analyzed.

Stability	Measures
Reserve Duties	Overtime
Cancellations of y _{wds}	Cancellations of y _{wds}
$\sum_{w \in W} \sum_{s \in S \smallsetminus s^*} c_{wds}$	$\sum_{w \in W} \sum_{s \in S \setminus s^*} c_{wds}$
• Cancellations of c_{wds}^r	• Cancellations of prescheduled overtime
$\sum_{w \in W} \sum_{s \in S \setminus s^*} y_{wds}^r$	$\sum_{w \in W} [(s_{wd} + f_{wd}) - (s'_{wd} + f'_{wd})]$
• Conversions of reserve duties to working duties	• The usage of the prescheduled overtime
$\sum_{w \in W} \sum_{s \in S \setminus s^*} [y'_{wds} - (y_{wds} - c_{wds})]$	$\sum_{w \in W} s'_{wd} + f'_{wd}$
• Understaffing tasks	• Understaffing tasks
$\sum_{j\in J_d} n_j$	$\sum_{j\in J_d} n_j$

Table 8: Stability Measures

Robustness

Incorporating reserve duties leads to a personnel roster that represents a higher-than-minimal cost in the tactical scheduling phase but leads to the advantage that the cost increase in the operational allocation phase may be within limits due to the increased robustness (Maenhout & Vanhoucke, 2010). In conformity with (Ingels & Maenhout, 2015), the planned and actual performance of the timetables will be evaluated to assess the robustness created by incorporating reserve duties. Table 9 indicates which cost will be incorporated in the total assignment cost under evaluation for both reserve duties and overtime. To assess the efficiency in terms of human resources, the conversion rate for the different capacity buffers will be analyzed.

Robustness Measures					
Reserve	e duties				
Planned Performance	Actual Performance				
Planned cost	Actual cost				
* Understaffing cost	* Understaffing cost				
Understaffing tasks	Understaffing tasks				
Understaffing reserve duties					
* T-t-1	* T-4-1				
Total assignment cost	lotal wage cost				
Regular duty assignment cost	Regular duty wage cost				
Reserve duty assignment cost	Reserve duty wage cost				
	Duty cancellation cost				
• Hiring cost	• Hiring cost				
Scheduled	l overtime				
Planned Performance	Actual Performance				
 Planned cost 	Actual cost				
* Understaffing cost	Understaffing cost				
Understaffing tasks	Understaffing tasks				
Understaffing scheduled overtime					
*Total assignment cost	* Total wage cost				
Regular duty assignment cost	Regular duty wage cost				
Cost of prescheduled overtime	Cost of overtime utilization				
	Duty cancellation cost				
• Hiring cost	• Hiring cost				

 Table 9: Robustness Measures

11 Computational results

This chapter includes the results obtained by solving the models from Chapters 6, 7, and 8. The resulting rosters will be used as a benchmark when comparing them to the results from the operational allocation phase. The first section describes the personnel roster from the integrated strategic staffing and tactical scheduling phase. Section 11.2 describes the evaluation of the averaged results from applying the operational allocation phase on the baseline rosters. The commercial software package Gurobi Optimization, LLC (2023) was used to solve the MIP models.

11.1 Resulting personnel timetables

11.1.1 Baseline personnel roster

The staffing plan derived from the baseline model (Chapter 6) for the first 10 staff members can be found in Table 10. The rosters for the other workers can be found in Tables 23 to 26 in Appendix A. The tables display the tasks each worker is assigned for each day of the planning period. Table 11 shows the color coordination used in the tables.

	Weekly Shift Roster Baseline - Part 1								
Worker	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7		
1	25,423	65 , 293	37,581	599,72	650,239				
2	512,179			463	299,659		211,227		
4	53	73		185 , 197	164		716,329		
6	513,445		381, 394	249,305	637, 349		115 , 714		
				, 308					
7	357, 531			266,360	638 , 119	680,112	337		
					, 316				
8	69,526		174		74	242 , 452	698 , 495		
10	13	183	142,578	610		686 , 436			
13		182	188 , 265	303,615	631		42		
14		190	565	591	657,345	145			
15		246,311	366, 372	313,371		50	189,230		

Table 10: Weekly Shift Roster Baseline - Part 1

Color Coordination Shift Rosters					
Early	Late	Night	Day off		

Table 11: Color Coordination Shift Rosters

It should be noted that constraints (8) and (9) from the model were relaxed to solve the problem since otherwise workers were scheduled to a working shift without being assigned to tasks in that shift. Given the fact that the objective function minimizes the staff size, the model will try to minimize the number of days off but still respect a minimum of 2 days off per week.

From the tables, it can be inferred that there is a total staff size of 96 workers. This implies that 54 workers were not assigned to any working shifts in the planning period, and are therefore not considered as part of the total workforce size. The total number of shift assignments per day is indicated in the table below, leading to an aggregated assignment count of 464. Note that the assignments for a day off are not included in this total sum.

Number of assignments per day						
	Early	Late Shift	Night	Total		
	Shift		Shift			
Day 1	16	28	27	71		
Day 2	22	19	16	57		
Day 3	18	18	23	59		
Day 4	23	22	31	76		
Day 5	25	28	18	71		
Day 6	20	21	18	59		
Day 7	27	22	22	71		

Table 12: Number of Assignments per day in the Baseline Personnel Roster

The set of maximal cliques *C* contains 197 different sets of overlapping tasks K_c , which can be found in Appendix B. The different tables hold the job IDs of the overlapping tasks. It should be noted that all cliques were determined for each shift $s \in S \setminus s^*$ on each day $d \in D$ separately. For this reason, the cliques of the different days over the planning period are separated by a line in the tables. K_{173} , a clique determined for the night shift on day 4, includes the largest amount of tasks, namely 31. Consequently, a minimum of 31 workers is required to perform all the tasks in the night shift on day 4. Looking at Table 12, we see that this lower bound of workers is satisfied. On average, K_c includes 19 tasks.

The objective value of the baseline roster, which minimizes the wage cost and the preference penalty cost, is equal to 21 133,6.

11.1.2 Personnel roster with proactively scheduled reserve duties

The model described in Section 7.1 adds a capacity buffer of reserve duties to the baseline roster (Chapter 6). Therefore, reserve duties are introduced but are not assigned to tasks yet. This reserve capacity can then be used in the operational allocation phase to cope with unexpected interruptions. Different sizes for the buffers were examined. The numbers of reserve duty assignments per buffer size are indicated in Table 13. Keep in mind that the buffer ratio is multiplied by the assigned staff size for each shift on each day and is thus calculated for each combination of shift $s \in S \setminus s^*$ and day $d \in D$.

Staff Sizes Reserve Duties							
	5 %	10 %	15 %	20 %	25 %	30 %	35 %
Number of reserve duty assignments	35	48	68	93	121	139	163
Total staff size	105	107	111	116	121	124	129

Table 13: Staff Sizes Reserve Duties

It should be noted that reserve duties can be allocated to all workers, there are no workers who receive only reserve duties and no working shift duties in the planned schedule. For example, with a buffer ratio of 25%, 121 reserve duty assignments are divided over a total workforce of 121. Table 14 shows the planned roster for 5 workers when a buffer size b of 25% is used. Worker 1 is assigned to 4 consecutive late working shifts on the first 4 days, and on day 5 he/she is assigned to a late shift as a reserve worker. He/she is not yet assigned to specific tasks, this will only be done in the operational allocational phase if the reserve duty assignment is converted to a working assignment. The minimum rest time of 11 hours is still respected when assigning reserve duty assignments. Table 15 indicates the color coordination used in the roster for the reserve duties. The same color coordination as in Table 11 is used for working shift assignments.

Weekly Shift Roster with Reserve Duties							
Worker	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	154, 310	173, 458	118, 572	218			
3	532	289	226		347		
4	222	446, 552			93, 655	36, 685	210, 696
5	139	73, 537				191	42
6			264, 394	295, 305,	419, 663		
				308			

 Table 14: Weekly Shift Roster with Reserve Duties

Color Coordinati	ion Shift Rosters w	vith Reserve duties
Early	Late	Night

Table 15: Color Coordination Shift Rosters with Reserve Duties

Of course, scheduling additional reserve duties on top of the required staffing constraints comes at a cost. The total costs for the different scenarios are shown in Figure 10. The augmentation in the total cost is due to the larger workforce size, the reserve duty assignments, and the preference score of workers for the shifts they are assigned to as reserve workers. We scheduled as many regular and reserve workers as needed to fulfill both the requirements concerning tasks and reserve duties. Consequently, the understaffing costs are equal to zero.



Total Cost - Reserve Duties

Figure 10: Total Cost Reserve Duties

11.1.3 Personnel roster with proactively scheduled overtime

A baseline model including a combination of resource and time buffers in the form of proactively scheduled overtime was described in Section 8.1. The outcome of the model for a buffer of a total of 6300 minutes, i.e. 105 hours, of overtime divided over 35 assignments is shown in Table 17. This buffer size is attained by assigning 5% of the assigned staff per shift overtime of 3 hours. Since the durations of the tasks are rather long, we need to assign a large amount of prescheduled overtime to use in the operational allocation phase. The overtime can be assigned in 2 forms, namely as a prior shift extension or a subsequent shift extension. The color coordination used to elucidate the distinction in the roster is shown in Table 18.

In order to distribute the overtime fairly among the staff, we decided to assign each employee the same amount of overtime. In addition, a worker can be scheduled to overtime at most 2 times a week and he/she is assigned to either a prior or subsequent shift extension per shift assignment. Including this additional buffer comes at a cost. The cost for different values of b and the corresponding amount of overtime each employee is assigned to is stated in Table 16.

Total Costs - Overtime							
	b = 5 %	b = 10 %	b = 15 %	b = 20 %	b = 25 %		
3 hours	21724,53	21943,60	22281,10	22753,60	23175,48		
3,5 hours	21822,66	22078,60	22472,35	23023,60	23515,79		
4 hours	21921,10	22213,60	22663,60	23293,60	23856,10		

Table 16: Total Costs - Overtime

Given the fact that the set B_s assures that the time between 2 consecutive is at minimum 16 hours and that a worker can be assigned to a maximum of 240 minutes or 4 hours of overtime per day, no additional workers needed to be hired to fulfill the requirements regarding overtime R_{ds}^o . Resulting in a total cost that only adds the cost of the proactively scheduled overtime on top of the cost obtained by solving the baseline model (Section 11.1.1).

	Day 7	156	337		712	16	247,	256
0	Day 6	668	281,	346	688	95	359,	277
time								_
luled Over	Day 5	23	236		321		241,	275
ched								
actively Sc	Day 4	603, 72	606		601	46	181	
Pro:								
ith	~							
oster w	Day 3		168		566	51	109	
ft R								
/eekly Shi	Day 2					19	124	
5								
		3						
	Day 1	69, 51						
	Worker	12	13		14	15	19	

Table 17: Weekly Shift Roster with Proactively Scheduled Overtime

Color Coordination Shift Rosters with Overtime	Prior Shift Extension Subsequent Shift Extension
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Table 18: Color Coordination Shift Rosters with Overtime

11.2 Evaluation

In this section, we evaluate the proactive inclusion of reserve duties in terms of both stability (Section 11.2.1) and robustness (Section 11.2.2). The evaluation will be solely performed on the model regarding the proactive inclusion of reserve duties. The model with the proactively scheduled overtime was not applicable to the database used in this dissertation since the average duration exceeds the maximal allowed overtime. As such, almost (none) of the understaffed tasks could be handled by the scheduled overtime. This led to an actual performance that was always worse than the planned performance.

11.2.1 Stability

The results in terms of the stability measures discussed in Section 10.3 are represented in Table 19. These results were averaged to get a global view of the stability of the actual personnel timetables.

Figure 11 shows the adaptations made in the operational allocation phase to the originally planned roster in order to make the schedule feasible and/or workable again. A personnel roster can be adapted by canceling reserve and regular duties and by converting reserve duties to working duties. The graph shows that the number of cancellations of regular duties stays constant over the different buffer sizes. This is the result of the fact that we enforced that when duties need to be canceled, the reserve duties are canceled first. Cancellations were also made when an assigned worker was not available anymore at the start of the shift. The proportion of the canceled reserve duties in the total adaptations undergoes exponential growth, while the conversion of reserve duties grows rather steadily. Inferring from this, we can state that the conversion rate is very low for large buffer sizes.



Figure 11: Changes to planned rosters

The larger the fixed buffer ratio b, the lower the understaffing of tasks (Figure 13) but the higher the cancellations of reserve duty assignments (Figure 12) and the overstaffing of shifts. The ability to cope with unanticipated disruptions enlarges gradually by incorporating additional reserve duty assignments but is detrimental to the number of cancellations. In light of this, a trade-off between the objectives of minimizing the understaffing of tasks and the overstaffing of shifts, which are inefficient in terms of cost and resource utilization, has to be made.



Reserve Duty Cancellations

Figure 12: Reserve Duty Cancellations



Figure 13: Task Understaffing

When we look at the computational results on a shift level, we get some interesting insights. Figure 14 indicates the task understaffing levels over the different buffer sizes for the early shift on day 1 and the late shift on day 7. We chose these particular shifts because they have respectively the smallest and largest amount of tasks assigned per employee for that shift. In the early shift on day 1, a worker is assigned to 1,25 tasks on average, while workers assigned to the late shift on day 7 have to finish 2,72 tasks on average. The fact that more tasks can be assigned per worker in the later results from the fact that the average duration of the tasks in the late shift on day 7 is shorter than the average duration in the early shift on day 1.



Understaffing - Shift Level

Figure 14: Understaffing on shift level

This graphical representation reveals that shifts with shorter tasks solve the task understaffing problem faster. Consequently, the conversion of reserve duties has more effect since he/she can be assigned to more tasks of short duration in comparison to when he/she is deployed in a shift with tasks of longer duration. However, this depends on the weight each task was given. In this dissertation, each task was given the same priority.

In terms of stability, we can conclude that the effectivity or the ability to minimize the understaffing of tasks under uncertainty is already significant by incorporating even a small capacity buffer. This ability grows even more with bigger buffer sizes but at a decreasing pace and with a fast-growing amount of changes to the originally planned timetable.

	Stability Measures - Results					
	Baseline	b = 5 %	b = 10 %	b = 15 %		
Cancellations of reg-	26,95	26,15	27,17	27,96		
ular duties						
• Cancellations of re- serve duties		20,48	29,05	43,54		
• Conversions of re- serve duties		14,52	18,95	24,46		
Overstaffing shifts	26,95	48,24	57,50	69,93		
• Understaffing tasks	32,50	19,28	16,34	13,09		
	b =20 %	b = 25 %	b = 30 %	b = 35 %		
• Cancellations of reg- ular duties	27,70	26,74	27,87	28,42		
• Cancellations of re- serve duties	66,71	92,81	109,87	129,16		
• Conversions of re- serve duties	26,28	28,19	29,09	33,86		
Overstaffing shifts	85,41	112,49	137,74	157,57		
• Understaffing tasks	6,07	3,58	1,92	1,20		

Table 19: Stability Measures - Results

11.2.2 Robustness

Whereas stability is primarily concerned with the achievement of minimal task understaffing without many disruptions to the originally planned personnel roster, robustness is interested in achieving this objective while optimally using the available resources of the organization. These resources are staff and money, and as such the efficiency of the capacity buffer, in the form of reserve duties, and the total assignment cost will be examined in this section. The computational results of the planned and actual performance of the personnel timetables can be found in Tables 20, 21, and 22.

The efficiency of the personnel resource can be measured by the conversion rate, i.e. the number of converted reserve duties in relation to the total number of planned reserve duties. The conversion rates presented in Figure 15 indicate that with growing buffer sizes, the conversion rate, and thus the efficiency of personnel resources drops. Including more reserve duties on top of the staffing requirements in the baseline results in more ability to cope with uncontrollable factors, but undermines the resourceful use of the reserve duties.



Conversion Rate

Figure 15: Conversion Rate

Since this utilization ratio gives an indication of how well the reserve duties were planned in the integrated strategic staffing phase and the tactical staffing phase, we can state that the larger the buffer size, the more attention should be paid to strategically planning the reserve duties. The increasing number of overstaffing in Table 19 also indicates this misallocation of staff members. The second type of resource under investigation is monetary assets. This financial resource is expressed in terms of the planned and actual performance, more specifically the objective functions in respectively the tactical and operational phases.

Figure 16 shows us that the actual performance starts to outperform the planned performance with the inclusion of a capacity buffer of 30%, which is rather high. This is due to the fact that we assigned the cost for understaffing tasks a very high weight. In this way, we prioritize meeting customers' demands at all times. However, by incorporating a buffer of only 5% the number of understaffed tasks is already reduced by 40% (Figure 13). With the inclusion of a buffer of 20% or 25% the planned and actual performance lay close to each other. Even though the total assignment costs are slightly higher for these capacity buffers, they are worth noting since they reduce the number of understaffed tasks significantly.

The higher we set the fixed buffer ratio b, the more the actual performance outperforms the planned performance. But this trend can't go on indefinitely, there will always be a small amount of understaffed tasks that can't be solved. The further incorporation of reserve duties will only lead to more cancellations and thus a higher cost. Consequently, there is a trade-off between the total incurred assignment cost and the priority of ensuring customer satisfaction. The fewer customer orders one wants to cancel, the higher the assignment cost will be.





Figure 16: Actual vs Planned Performance

When we break down the total assignment cost into different pieces, we can state that the higher the capacity buffer, the higher the wage and cancellation cost will be. A high buffer can limit the number of understaffed tasks significantly by converting reserve duties to working assignments when disruptions due to uncertainty occur. Despite that, this leads to a low conversion rate and consequently a high cancellation cost.

A second cost that takes up a big part of the total assignment cost is the preference penalty cost. In the tactical scheduling phase, workers are scheduled according to their preferences regarding the shift they are assigned to. However, in the operational allocation phase, working duties get canceled and workers are assigned a day off while their preference penalty score is higher with regard to the preference penalty score of the shift they were originally assigned to.

Note that since we use the fixed reactive mechanism, which assumes that an employee's schedule cannot be changed in the short term, the flexibility introduced is rather limited. The usage of the adjustable reactive mechanism offers more flexibility because workers can change shifts in the planning period if necessary. To exclude the reactive impacts of the strategy, the use of swaps was excluded from this dissertation.

A manner in which the flexibility of timetables with proactively assigned reserve duties can be increased is by scheduling reserve duties that have sufficient skills for the tasks. In this way, the reserve duty can be conversed and assigned to most of the tasks of that shift. Given that we work with a heterogeneous workforce, not all workers master the same set of skills. It is important to note that this is not the same as using employee substitutability as a proactive strategy (Section 3.3). Reserve workers are planned as backups to ensure the continuity of the operations in an organization, while employee substitutability focuses on cross-training personnel such that duties can be swapped between workers based on the skills they possess. By assigning (reserve) workers that possess multiple skills, no tasks should be canceled because there was no reserve worker available to step in for the assigned worker that contains the required skills but was not available any more on the day of operation to perform the task. Consequently, the quality of the reserve duty depends on the multi-skilling level of the worker.

	Planne	ed Performance		
	Baseline	b = 5 %	b = 10 %	b = 15 %
Wage cost				
Workforce size	40 * 96 = 3840	40 * 105 = 4200	40 * 107 = 4280	40 * 111 = 4440
Assigned shift duties				
Regular workers	30 * 464	30 * 464	30 * 464	30 * 464
Reserve workers	- 12020	+ 20 * 35	+ 20 * 48	+ 20 * 68
	= 13920	= 14620	= 14000	= 15280
Preference penalty score	2 * 1686, 8	2 * 1765.07	2 * 1797, 1	2 * 1840.24
Total Cost	21133,60	21990,14	22754,02	23400,48
	b = 20%	b = 25 %	b = 30 %	b = 35 %
Wage cost				
• Workforce size	40 * 116 = 4640	40 * 121 = 4840	40 * 124 = 4960	40 * 129 = 5160
• Assigned shift duties				
Regular workers	30 * 464	30 * 464	30 * 464	30 * 464
Reserve workers	+ 20 * 93 = 15780	+ 20 * 121 = 16340	+ 20 * 139 = 16700	+ 20*163 = 17180
Preference penalty score	2 * 1914,26	2 * 1972,33	2 * 2006,33	2 * 2033,58
Total Cost	24248,52	25124,66	25672,66	26407,16

Table 20: Planned Performance	e
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	Actual Perf	formance - Part	1	
	Baseline	b = 5 %	b = 10 %	b = 15 %
Wage Cost				
• Hiring cost	40 * 96 = 3840	40 * 105 = 4200	40 * 107 = 4280	40 * 111 = 4440
• Assigned shift duties				
Regular workers	30 * 437,05	30 * 436,85	30 * 436,18	30 * 433, 96
Reserve workers		35 * 14.52	35 * 18,95	35 * 24,46
	= 13111,49	= 13613,7	= 13748,65	= 13874.9
Preference penalty cost	2* 1702,14	2* 1791,48	2* 1809,66	2* 1844,66
Understaffing tasks	85 * 32,50	85 * 19,29	85 * 16,34	85 * 13,09
Cancellations of regular duties	25 * 26,95	25 * 26,15	25 * 27,17	25 * 27,96
Cancellations of re- serve duties		15 * 20,48	15 * 29,05	15 * 43,54
Total Cost	23792,02	23997,26	24151,87	24468,97

Table 21: Actual Performance - Part 1

	Actual Perf	ormance - Part	2	
	b = 20 %	b = 25 %	b = 30 %	b = 25 %
Wage cost				
• Hiring cost	40 * 116 = 4640	40 * 121 = 4840	40 * 124 = 4960	40 * 111 = 5160
• Assigned shift duties				
Regular workers	30 * 435,48	30 * 435,26	30 * 436,13	30 * 435,58
Reserve workers	35 * 26,29	35 * 28,2	35 * 29,10	35 * 33,86
	= 13984,55	= 14044,80	= 14102,40	= 14252,50
Preference penalty cost	2* 1927,86	2* 1981,48	2* 2014,36	2* 2041,65
Understaffing tasks	85 * 6,07	85 * 3,58	85 * 1,92	85 * 1,20
Cancellations of regular duties	25 * 27,70	25 * 26,74	25 * 27,87	25 * 28,42
Cancellations of re- serve duties	15 * 66,71	15 * 92,81	15 * 109,87	15 * 129,16
Total Cost	24289,37	25202,71	25242,71	26245,70

Table 22: Actual Performance - Part 2

11.2.3 Conclusions

To conclude our analysis on stability and robustness, we can state that the stability of a personnel timetable is reached faster, i.e. a significant amount of understaffed tasks can be solved with the incorporation of a small capacity buffer. The more we increase the buffer size, the more adaptations have to be made to the personnel roster. This has hazardous effects on employee satisfaction.

When we look at the robustness of timetables with proactively assigned reserve duties, we can assert that a bigger quantity of reserve duties needs to be included in the originally planned schedule in order to outperform the planned performance in the operational allocation phase. Even though, a big number of backup workers need to be included in the empirical analysis made in this dissertation until actual performance outperforms the initially planned performance, incorporating even a smaller amount of reserve duties already leads to beneficial effects on the total sum of understaffed tasks.

In order to optimize the use of reserve duties, they should be strategically scheduled in the initial timetable. We found that when incorporating reserve duties in shifts with tasks with shorter duration the number of understaffed tasks drops faster. This is due to the fact that a worker, reserve or regular, can perform more tasks during this shift. Another recommendation is to use reserve duties that master a large set of skills, accordingly when their reserve duty has conversed to a working assignment they can perform multiple tasks and no tasks will be canceled because there was no one available with the required set of skills.

12 Conclusions and recommendations

12.1 Recommendations

Based on the analysis performed on the empirical study conducted in this dissertation, we can conclude that including even a small capacity buffer is beneficial to cope with the uncertainty faced in the stochastic operational environment organizations operate in. Given the fact that unanticipated events can make the originally planned time infeasible, organizations should always include even a small buffer in their personnel schedules in order to cope with uncertainty.

We recommend placing the capacity buffer strategically, i.e. in shifts where they can have the most impact. As such, the wage and cancellation cost of reserve duties can be minimalized while the impact they have on the total amount of understaffed tasks can be maximalized. Another benefit of this is employee satisfaction since fewer reserve duties will be canceled.

Moreover, the quality of the buffer can have a significant impact on the objective. By assigning reserve duties that master a large set of skills, they can be backups of employees with different skills. As such, the reserve duties can be utilized for different sets of tasks.

12.2 Conclusions

In this dissertation, the impacts of the inclusion of proactive strategies on the stability and robustness of a personnel timetable were examined. The personnel scheduling process and the different proactive (and reactive) strategies were described in the literature review. In order to investigate the impact on personnel rosters, we build the baseline models with and without the inclusion of reserve duties or overtime for the integrated strategic staffing and tactical scheduling phase. In addition, the models to test in the operational allocation phase were set up. By making use of a discrete-event simulation, we replicated the stochastic environment and tested how well the models could cope with the faced interruptions.

To give an answer to the posed research questions, we can state that for small capacity buffers the stability of the personnel timetables still holds. By incorporating more reserve duties, we get a more robust timetable but this comes at a cost of course. Depending on the value one gives to the objective of maximizing customer satisfaction, a fitting buffer size needs to be incorporated in the integrated strategic staffing and tactical scheduling phase.

Not only the buffer size has an impact on the performance of the timetables in the stochastic operational environment, but the positioning and the quality of the reserve duties also play an important role. Placing the reserve duties in shifts where they can be assigned to multiple tasks if needed, can reduce the amount of understaffed tasks significantly. The quality of the reserve workers, in terms of the set of skills they master, needs to be sufficient to make sure that they can be assigned to as many tasks as possible for which they possess the required skills.

The contribution of this dissertation is the fact that the inclusion of reserve duties is tested on the personnel task and shift scheduling problem in an empirical study with a rather large problem size. The models to incorporate overtime in timetabling were set up, but have yet to be tested with regards to robustness and stability.

12.3 Limitations and future research

The scope of this dissertation was to analyze the robustness and stability of timetabling applications. We used the integrated personnel shift and task scheduling model to build the personnel rosters. We introduced 2 types of uncertainty, namely the uncertainty of demand and the uncertainty of capacity. However, to replicate the stochastic operational environment, we suggest inserting the uncertainty of arrival into the research on the robustness and stability of timetabling applications.

Another limitation of the research done in this dissertation is the fact that the planned task had a duration with rather long duration. As such, sometimes a worker was only assigned to 1 task per shift and making it a shift scheduling problem, rather than a task and shift scheduling problem. Analyzing the impact on idle time could be a very interesting approach when making use of a task set containing tasks of shorter duration.

Furthermore, when the demand for staff changed in the operational allocation phase due to a change in the staffing requirements, we duplicated or deleted a task. It might be interesting for further research to build new tasks, with their own duration, start and finish times, set of skills, etc.

With regard to the investigation of different proactive strategies, we set up the models for both reserve duties and overtime. However, we only did the evaluation based on the model for reserve duties. An additional investigation should be made into the inclusion of other proactive strategies and even in combinations of multiple strategies. To finalize, this dissertation was only a small fraction of the research on improving robustness and stability by including proactive strategies in the personnel scheduling process. Subsequent exploration of the subject in terms of additional strategies, measures, and different contexts is still required.

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		Weekly	Weekly Shift Roster Baseline Part 2				
Worker	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
19	135		226	607		12	250,386
20	335,447	207		598,92	663,419		161
22	297,434		576	619,327		68	733, 473
				, 468			
23	32,518	535,319		600		679	728
24		547,446	570	589	431,667		732,479
25	516		8		152	97	100
26	524		129	84	644		
27		534,493		181	59	83	718,234
28	108	184	75,433,	602,358			709,338
			585				
31	507	553,333		613 , 469			155 , 719
32	76		58,583	612,214	641		107
35	144	173, 193		596		21	703,318
36	159	136	566	623,350			695 , 481
37	510		320		639		11
39	104 , 382		569	617		94	706 , 116
40	504	231,556			63	673,402	731
43	529,487		109	593, 79	628 , 323	301	
44	204		43	64	632,93	690,410	
45	532		560	590	648 , 236	672 , 453	
46	154 , 177	5,555		294,347	661,341	200	
47		14,538	575		2	96	213,723
48	508		51	61	148 , 656		127
52	87	279,374		269,273		191	16
53		151	114	352		678	45,705
54	501		201		634,430	689	725
56		22,243	457,459	592		149,403	198

A Appendix A: Results baseline Rrster

Table 23: Weekly Shift Roster Baseline Part 2

		Weekly	Shift Rost	ter Baselin	e Part 3		
Worker	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
62	334 , 409	377,426		285,383	647	101 , 282	
63		215		91,449	629 , 392		133
					, 443		
64			153	82,262	106 , 651	385,448	734
66	505	356, 450	574		220,241		729, 422,
							424
67	500,28	541	138	611			713, 317
68		549,458	168	163,614	640		710, 342,
							411, 429
69		542,315	564,471	178	321		71,704
70	511	160		146	23,633		700,288
72	514		17,568	110	660,340		196 , 292
75	89		261 , 298	364,378	131 , 406	95	
			, 367				
77		41	577		7	671	702
79		60,432	586,496		31 , 263	128	712
80	397	548	559	626 , 465		36,685	
81	290, 395	217, 291		4,248	187 , 258		711,210
					, 384		
85	509		1	302	654 , 280	44 , 408	
					, 348		
86	502	228		55,603	662,470	195	
87	270, 396	543,441		34		435,483	
89	268,414			46	425	362,415	390, 393
92	77	552			219,399	134	88
94		70	561	595	353,666	346 , 454	

Table 24: Weekly Shift Roster Baseline Part 3

		Weekly	Shift Rost	er Baselin	e Part 4		
Worker	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
95	515		172 , 582	27,618,	664		365
			, 401	388			
96	522		579,339	608		225	720
98	503 , 486		563,171	223			701
			, 451				
99	521		584,476	354		368,400	257
100	523,332		572	218	627,440		727
	, 488						
101	9	252,373	558,26	622,467		692,407	
103		540		90	49		387
104	520,203	545,491	461,472		48	684,494	
105		56,375		244, 369	206	669 , 235	708
						, 260	
107	330		573,221		57	675 , 113	696 , 284
							, 314 ,
							418
109	325,530			24,609	655 , 464		208,240
112		52, 391	567,118		275	245	162 , 412
							, 442
114	29	289,497	571,355		259		697,35
115	506,310	539,117		604,306		676 , 274	715,398
	, 417			, 462			, 427
118	102			597,344	645,312	176 , 693	717,216
120	10	19	157,324		636	267	
			, 466				
121	498,336	550,478		165 , 621		62,681	707
122	238		482,484		283	277	

Table 25: Weekly Shift Roster Baseline Part 4

		Weekly	Shift Rost	er Baselin	e Part 5		
Worker	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
125	519,166			143 , 229	205 , 649	343 694	724
126	139		251,363	105	271,370		86
					, 413		
128		123	81,361	47, 295	132	687	
130	85	67			331,665	677	730,416
					, 428		
132	517,438		209		20		125
133		124	147	606		674	326 , 726
134		551	557,78	103 , 620	653		421 , 460
135	199	546		616		276 , 404	192 , 721
							, 379
136	499,309		141 , 264	492	646 , 477	682	
	, 485		, 405				
137	222			18,625	278,307	668,39	40,272
138		544,489			389	99,359	137
142	194	111 , 554		322,376	254,304		15
	_			, 624			
143	38			237	652	688	
144	98,528,		580,475	588,286	643 , 455	281,691	
	439				, 456 ,	, 490	
					480		
145			175 , 180	66	630,130		186
147	158 , 527	537,420		587,54	635,232		253
148		255,300	380,444	126	122,437		156
149	140	536, 296	562,120		3		247,256
150		121 , 224		6	202,658	683	722

Table 26: Weekly Shift Roster Baseline Part 5

B Appendix B: The set of maximal cliques C

	The set of maximal cliques C - Early Shift
K_1	9 10 13 33 85 87 108 140 159 194 204 270 290 382
K_2	9 10 13 33 85 87 104 108 140 159 194 204 395 396 397 409
<i>K</i> ₃	9 10 13 33 85 87 104 108 194 204 334 395 396 397
K_4	19 41 121 123 124 136 151 182 183 184 190 212 243 246 252 255 291 374 375 377 391
K_5	19 41 70 121 123 124 136 151 182 183 184 190 212 243 246 252 255 291 374 375 377
K_6	19 22 41 52 70 121 123 124 136 151 182 183 184 190 212 246 252 255 291 374 375 377
K_7	19 22 41 52 70 121 123 124 136 151 182 183 184 190 212 252 255 291 311 375 377
K_8	19 22 41 52 70 121 123 124 136 151 182 183 184 190 212 279 291 311 375 377
K_9	19 22 41 52 70 121 123 124 136 151 182 183 184 190 212 279 311 373 375 377
<i>K</i> ₁₀	19 22 41 52 56 70 121 123 124 136 151 182 183 184 190 212 279 300 311 373 377
<i>K</i> ₁₁	19 22 41 52 56 70 121 123 124 136 151 182 183 184 190 212 279 300 311 373 426
K_{12}	19 22 41 52 56 70 123 124 136 151 182 183 184 190 224 279 300 311 373 426
<i>K</i> ₁₃	19 22 41 52 56 70 123 124 136 151 170 182 183 190 224 279300 311 373 426
K_{14}	19 22 41 52 56 70 123 124 151 170 182 183 190 217 224 279 300 311 373 426
<i>K</i> ₁₅	1 43 51 109 129 153 180 188 201 209 361 363 367 372 380 405
<i>K</i> ₁₆	1 8 43 51 109 129 153 180 188 201 209 363 367 380
<i>K</i> ₁₇	1 8 43 51 81 109 129 153 180 188 201 209 261 363 380
<i>K</i> ₁₈	1 8 43 51 81 109 129 153 180 188 201 209 261 264 363 366 381 444
K_{19}	1 8 43 51 81 109 129 153 175 201 251 261 265 366 394 444
<i>K</i> ₂₀	1 8 43 51 81 109 141 153 175 201 251 265 298 366 394 444
<i>K</i> ₂₁	6 46 61 64 66 84 105 126 146 181 185 248 249 262 269 285 295 360 364 369
K_{22}	6 46 61 64 66 84 105 126 146 181 185 248 249 262 269 285 295 360 369 371
<i>K</i> ₂₃	6 46 61 64 66 84 90 105 126 146 181 185 248 249 262 269 285 295 371
K_{24}	6 46 61 64 66 84 90 105 126 146 181 185 248 262 269 285 295 371 378
K_{25}	4 6 46 47 61 64 66 84 90 105 126 146 181 185 269 285 371 378
K_{26}	4 6 46 47 61 64 66 82 84 90 105 126 146 181 185 244 266 273 285 302 305 371 378
K_{27}	4 6 46 47 61 64 66 82 84 90 105 126 146 181 185 244 266 273 302 305 313 378
K ₂₈	4 6 46 47 61 64 66 82 84 90 105 126 181 185 244 266 273 302 305 313 378 383
<i>K</i> ₂₉	4 6 46 47 61 64 66 82 84 90 105 181 197 244 266 305 313 378 383
<i>K</i> ₃₀	4 6 46 47 61 64 66 82 84 90 105 181 197 266 308 313 378 383

Table 27: The set of maximal cliques C - Part 1
	The set of maximal cliques C - Early Shift		
<i>K</i> ₃₁	3 7 20 48 57 59 74 131 206 241 258 263 370		
K_{32}	2 3 7 20 31 48 49 57 59 63 74 131 152 206 241 258 283 389 399 413 425 437		
K_{33}	2 3 7 20 31 48 49 57 59 63 74 131 152 206 259 275 283 389 399 413 425 437		
K_{34}	2 3 7 20 31 48 49 57 59 63 74 131 152 206 220 259 275 283 384 389 399 425 437		
K_{35}	2 3 7 20 31 48 49 57 59 63 74 132 152 206 219 220 259 271 275 283 384 389 406 437		
K_{36}	2 3 7 20 31 48 49 57 59 63 74 132 152 187 206 219 220 259 271 275 283 389 406 437		
<i>K</i> ₃₇	2 3 7 20 31 48 49 57 59 63 74 122 132 152 187 206 219 220 259 271 275 283 389 425		
K ₃₈	21 50 68 83 94 95 96 128 134 191 242 359 400		
K_{39}	21 50 68 83 94 95 96 97 128 134 191 242 245 276 277 362 400 403		
K_{40}	12 21 50 68 83 94 95 96 97 99 128 134 191 242 245 276 277 362 368 403		
K_{41}	12 21 50 68 83 94 95 96 97 99 128 134 191 245 276 277 362 368 452		
K_{42}	12 21 50 68 83 94 95 96 97 99 128 134 149 191 245 277 368 404 415 452		
K_{43}	15 16 42 86 88 100 107 125 127 133 137 161 186 189 196 198 211 240 247 257 365 386		
	387 412		
K_{44}	11 15 16 42 86 88 100 107 125 127 133 137 161 186 189 196 198 211 240 247 250 253 257		
	387 393 412		
K_{45}	11 15 16 42 86 88 100 107 125 127 133 137 161 186 189 196 198 211 250 253 257 365 387		
	442		
K_{46}	11 15 16 42 86 88 100 107 125 127 133 137 161 186 189 196 198 211 250 253 256 365 387		
	442		
K_{47}	11 15 16 42 86 88 100 107 125 127 133 137 161 186 189 196 198 211 250 253 256 390 442		
K_{48}	11 15 16 42 86 88 100 107 125 127 133 137 186 189 196 198 227 253 256 390		
K_{49}	11 15 16 42 86 88 100 107 125 127 133 137 162 186 189 196 198 227 256 390		
K_{50}	11 15 16 42 86 88 100 107 125 127 133 162 186 196 208 227 230 256 390		

Table 28: The set of maximal cliques C - Part 2

	The set of maximal cliques C - Late Shift
<i>K</i> ₅₁	498 499 500 506 512 513 517 518 139 144 154 222 297 526 527 528 423
K_{52}	498 499 500 506 512 513 517 518 139 144 154 199 222 297 526 527 423
K_{53}	498 499 500 29 506 512 513 517 518 139 144 154 199 222 297 526 423
K_{54}	498 499 500 29 506 512 513 517 139 144 154 158 199 222 287 297 526 414 423 439
K_{55}	498 499 29 506 77 512 513 517 139 144 154 158 199 222 287 297 526 414 423 439
K_{56}	498 499 29 77 512 513 89 102 517 139 144 154 158 199 222 287 297 335 414 417 423 439
K_{57}	498 29 76 77 512 513 89 102 517 139 144 154 158 199 222 287 309 335 414 417 423 439
K_{58}	498 29 38 53 76 77 512 513 89 102 517 139 144 154 158 199 222 287 309 335 417 423
K_{59}	498 29 38 53 76 77 512 89 102 517 139 144 154 158 199 222 268 287 309 335 417 423
K_{60}	498 29 32 38 53 69 76 77 512 89 102 517 139 144 154 158 199 222 268 309 335 417 423
K_{61}	29 32 38 53 69 76 77 512 89 98 102 517 139 144 154 158 199 222 268 309 335 336
K_{62}	25 28 29 30 32 38 53 69 76 77 89 98 102 517 139 144 154 158 179 199 222 268 309 335
	336 434
K_{63}	25 28 29 30 32 38 53 69 76 77 89 98 102 139 144 154 158 179 199 222 268 309 335 336
	434 438
K_{64}	25 28 29 30 32 38 53 69 76 77 89 98 102 139 144 154 158 179 199 222 268 309 336 434
	438 447
K_{65}	25 28 29 30 32 38 53 69 76 77 89 98 102 139 144 158 177 179 199 222 268 309 336 434
	438 445 447
K_{66}	25 28 29 30 32 38 53 69 76 77 89 98 102 139 144 158 177 179 199 222 268 310 336 434
	438 445 447 485
K_{67}	535 536 537 538 539 543 547 549 551 552 293 554 555
K_{68}	535 536 537 538 539 543 547 193 551 552 293 554 555 458
K_{69}	535 536 537 538 539 543 547 193 207 551 552 293 555 432 458
K_{70}	536 537 538 539 543 111 547 193 207 551 552 293 432 458
K_{71}	536 538 539 543 73 111 547 193 207 551 552 293 432 458
K_{72}	536 538 539 60 540 67 543 73 111 547 193 207 551 552 293 319 458
K_{73}	536 538 539 60 540 67 73 111 547 193 207 551 552 293 319 420 441 458
K_{74}	14 539 60 540 65 67 73 111 547 193 207 551 552 319 420 441 458
K_{75}	5 14 60 540 65 67 73 111 117 547 193 207 551 552 296 319 420 441 458
K_{76}	5 14 60 540 65 67 73 111 117 173 207 551 296 319 420 441
K_{77}	5 14 60 540 65 67 73 111 117 173 207 551 296 319 441 446

Table 29: The set of maximal cliques C - Part 3

The set of maximal cliques C - Late Shift				
K ₇₈	557 558 562 563 567 568 572 573 577 578 580 581 582 324 583 584 585			
K ₇₉	558 562 563 567 568 572 142 577 580 581 582 324 583 584 585			
K ₈₀	558 58 567 568 114 120 572 142 577 221 580 581 582 324 584 585 451			
K ₈₁	558 58 567 114 120 572 142 577 221 581 324 401 433 451			
K ₈₂	26 58 78 567 114 120 572 142 577 221 581 324 401 433 451 475			
K ₈₃	17 26 37 58 78 567 114 120 572 142 577 221 324 401 433 451 475 476			
K ₈₄	17 26 37 58 78 567 114 120 572 142 577 221 401 433 466 475 476			
K ₈₅	17 26 37 58 75 78 114 120 572 142 577 221 401 466 475 476			
K ₈₆	17 26 37 58 75 78 114 118 120 142 171 577 221 401 466 475 476			
K ₈₇	17 26 37 58 75 78 114 118 120 142 171 172 221 466 475 476			
K ₈₈	17 26 37 58 75 78 114 118 120 142 157 171 172 221 475 476			
K ₈₉	587 588 593 594 597 598 599 603 604 605 609 612 218 618 620 621 624 625			
K ₉₀	587 588 593 594 597 598 599 603 604 605 609 143 612 218 620 621 624 625			
K ₉₁	587 588 593 594 597 598 599 603 604 605 609 143 612 218 620 621 376 625 388 449			
K ₉₂	587 588 593 594 597 598 599 603 604 605 143 214 218 620 621 376 625 388 449			
K ₉₃	587 588 594 597 603 605 91 143 214 218 620 376 388			
K ₉₄	587 34 594 603 91 103 143 214 218 376 388			
K ₉₅	24 34 80 603 91 103 143 214 218 376 388			
K ₉₆	24 27 34 54 80 603 91 103 143 214 218 306 376			
K ₉₇	18 24 27 34 54 55 72 79 80 91 92 103 110 143 165 214 218 233 286 306 344 376			
K ₉₈	18 24 27 34 54 55 72 79 80 91 92 103 110 165 214 218 229 233 286 306 344 376			
K ₉₉	18 24 27 34 54 55 72 79 80 91 92 103 110 165 214 218 229 233 286 322 344 462			

Table 30: The set of maximal cliques C - Part 4

The set of maximal cliques C - Late Shift			
<i>K</i> ₁₀₀	627 628 629 630 632 633 634 635 636 637 638 642 644 645 647 649 650 651 654 655 656		
	658 664 665		
<i>K</i> ₁₀₁	627 628 629 630 632 633 634 635 636 637 638 642 644 645 167 647 649 650 651 654 655		
	656 307 664 665		
<i>K</i> ₁₀₂	627 628 629 630 632 633 634 635 636 637 638 642 644 645 164 167 647 202 650 651 654		
	655 656 307 664 665		
<i>K</i> ₁₀₃	627 628 629 630 632 633 634 635 636 637 638 642 644 645 164 167 647 202 205 650 651		
	254 655 656 280 307 664 665		
K_{104}	627 628 629 630 632 633 634 635 636 637 642 644 148 645 164 167 647 202 205 650 651		
	254 655 280 307 664 665		
<i>K</i> ₁₀₅	627 628 629 632 633 634 635 636 637 642 130 644 148 645 164 167 647 202 205 650 651		
	254 655 280 307 664 665		
<i>K</i> ₁₀₆	627 632 634 635 642 130 644 148 645 164 167 647 202 205 650 651 254 655 280 307 392		
	664 665		
<i>K</i> ₁₀₇	627 632 634 635 642 130 644 148 645 164 167 647 202 205 650 651 254 655 280 307 316		
	392 664 428		
<i>K</i> ₁₀₈	627 632 634 635 642 130 644 148 645 164 167 647 202 205 650 651 254 280 307 316 392		
	664 428 464		
<i>K</i> ₁₀₉	627 23 632 634 642 130 644 148 645 164 167 647 202 205 650 651 254 280 307 316 323		
	392 428 464		
<i>K</i> ₁₁₀	627 23 642 130 644 148 645 164 167 647 202 205 650 651 307 316 323 348 428		
<i>K</i> ₁₁₁	23 642 130 644 148 645 164 167 647 202 205 650 651 307 316 323 348 428 443		
<i>K</i> ₁₁₂	23 93 642 130 644 148 164 167 647 202 205 650 651 278 304 312 316 323 348 428 430		
	440 443		
<i>K</i> ₁₁₃	23 93 106 642 130 644 148 164 205 650 278 304 312 316 323 348 430 440 443		
<i>K</i> ₁₁₄	23 93 106 119 130 644 148 164 205 650 278 304 312 323 348 440 443		
<i>K</i> ₁₁₅	23 93 106 119 130 644 148 164 205 232 239 278 304 312 331 348 349 443 474		
<i>K</i> ₁₁₆	668 669 673 675 676 677 678 679 680 681 685 686 169 267 282 301 690 385 692 408 693		
<i>K</i> ₁₁₇	668 673 675 676 677 678 679 680 681 685 686 169 260 267 282 301 385 408 693		
<i>K</i> ₁₁₈	668 673 676 677 678 679 680 681 686 169 260 267 282 301 407 408 693		
<i>K</i> ₁₁₉	668 36 673 677 678 679 680 681 113 686 169 260 267 282 301 407 408 410 693		
K_{120}	668 36 673 677 678 679 680 681 113 686 169 260 267 274 282 301 407 408 410 448		
K_{121}	668 36 44 681 113 686 169 260 274 301 402 407 410 448		
K_{122}	668 36 44 681 112 113 686 169 260 274 402 407 448		
<i>K</i> ₁₂₃	668 36 44 62 112 113 686 169 274 402 407 448		
K_{124}	36 44 62 101 112 113 686 169 274 402 407 448		
K_{125}	36 39 44 62 101 112 113 150 235 274 402 436 448		
<i>K</i> ₁₂₆	36 39 44 62 101 112 113 150 176 235 402 436 448		

Table 31: The set of maximal cliques C - Part 5

	The set of maximal cliques C - Late Shift				
<i>K</i> ₁₂₇	695 696 697 700 703 704 705 706 709 710 711 156 713 714 715 716 718 719 721 272 729				
<i>K</i> ₁₂₈	695 696 697 700 703 704 705 706 709 711 156 713 714 716 718 719 721 272 342 729 398				
	421				
<i>K</i> ₁₂₉	695 696 697 700 703 704 705 706 709 156 713 714 192 716 718 719 272 342 729 398 421				
<i>K</i> ₁₃₀	695 697 700 703 704 705 706 156 713 714 192 716 210 718 719 272 342 729 398 421				
<i>K</i> ₁₃₁	695 697 703 705 156 713 192 716 210 718 719 272 342 729 398 418 421				
<i>K</i> ₁₃₂	695 35 705 116 156 713 192 716 210 718 719 272 314 338 342 421 427				
<i>K</i> ₁₃₃	695 35 116 156 713 192 716 210 718 719 272 288 314 338 421 422 427				
<i>K</i> ₁₃₄	695 35 115 116 156 713 192 716 210 718 719 288 314 318 338 421 422 427				
<i>K</i> ₁₃₅	35 45 115 116 156 713 192 716 210 718 719 288 314 318 338 421 422 427				
<i>K</i> ₁₃₆	35 40 45 115 116 156 713 192 716 210 718 719 288 314 318 338 422 427 429 460				
<i>K</i> ₁₃₇	35 40 45 115 116 156 192 716 210 718 719 288 314 317 318 338 422 427 429 460 481				
<i>K</i> ₁₃₈	35 40 45 115 116 156 192 210 718 719 288 314 317 318 329 338 424 427 429 460 481				
<i>K</i> ₁₃₉	35 40 45 71 115 116 156 210 718 719 288 314 317 318 329 338 424 427 429 460 481				
<i>K</i> ₁₄₀	35 40 45 71 115 116 156 210 718 719 288 314 317 318 329 338 379 411 424 427 460 481				
<i>K</i> ₁₄₁	35 40 45 71 115 116 156 210 719 234 288 317 318 329 338 379 411 424 427 460 481				
<i>K</i> ₁₄₂	35 40 45 71 115 116 155 156 210 234 284 288 317 318 329 338 379 411 424 427 460 481				

Table 32: The set of maximal cliques C - Part 6

	The set of maximal cliques C - Night Shift
<i>K</i> ₁₄₃	501 502 503 504 505 507 508 509 510 511 514 515 516 135 519 520 521 522 523 238 524
	525 330 529 530 531 532
<i>K</i> ₁₄₄	501 502 503 504 505 507 508 509 510 511 514 515 516 135 520 166 521 522 238 525 330
	529 530 531 532
<i>K</i> ₁₄₅	501 502 503 504 505 507 508 509 510 511 514 515 516 135 166 521 522 203 238 525 330
	530 531 532 487
<i>K</i> ₁₄₆	501 502 504 505 507 508 509 510 511 514 515 516 135 166 521 522 203 238 525 330 530
	532 487 488
<i>K</i> ₁₄₇	501 502 504 505 507 508 509 510 511 514 515 516 135 166 521 522 203 238 328 532 487
<i>K</i> ₁₄₈	501 502 504 505 507 508 509 510 514 515 516 135 166 521 522 203 238 328 332 532 487
<i>K</i> ₁₄₉	501 502 504 505 507 508 509 510 514 515 516 135 166 521 522 203 238 325 328 332 532
<i>K</i> ₁₅₀	501 502 504 505 507 508 509 510 514 515 516 135 166 521 522 203 238 325 328 332 486
<i>K</i> ₁₅₁	501 502 504 507 508 509 510 515 135 166 521 522 203 238 325 328 332 357 486
<i>K</i> ₁₅₂	533 534 541 542 544 545 546 548 550 215 289 553 556 450
<i>K</i> ₁₅₃	533 534 541 542 544 545 546 548 215 228 289 553 450
<i>K</i> ₁₅₄	533 534 541 542 544 545 546 160 548 215 228 231 289 450
<i>K</i> ₁₅₅	533 534 541 542 544 545 546 160 548 215 228 231 289 356 478
<i>K</i> ₁₅₆	533 534 541 544 545 546 160 548 215 228 231 356 478 497
K ₁₅₇	533 534 541 544 546 160 548 215 228 231 315 333 356 478 491 497
K ₁₅₈	533 160 548 215 228 231 315 333 356 478 491 493 497
<i>K</i> ₁₅₉	160 548 215 228 231 315 333 356 478 489 491 493 497
K ₁₆₀	559 560 561 564 565 566 569 570 571 147 574 575 576 174 579 226 586
<i>K</i> ₁₆₁	559 560 561 564 565 566 569 570 571 138 147 574 575 576 174 579 226 320 459 472
K ₁₆₂	559 560 561 564 565 566 569 570 571 138 147 574 168 575 576 174 226 320 339 459 472
<i>K</i> ₁₆₃	559 560 561 564 565 566 569 570 138 147 574 168 575 576 174 226 320 339 355 459 472
	484
<i>K</i> ₁₆₄	559 560 561 564 565 566 569 570 138 147 574 168 575 576 174 226 320 339 355 457 472
	484 496
K ₁₆₅	559 560 561 564 565 566 570 138 147 574 168 575 576 174 226 320 339 355 457 461 484
	496
<i>K</i> ₁₆₆	559 560 565 566 570 138 147 574 168 575 576 174 226 355 461 471 482 496

Table 33: The set of maximal cliques C - Part 7	able 33: T	he set of	f maximal	cliques	С-	Part 7
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K ₁₁₇₇ 589 590 591 592 595 596 600 601 602 606 607 608 610 611 178 613 614 615 616 617 619 622 623 352 626 633 47 352 626 463 K ₁₀₆₀ 589 590 591 592 595 596 600 601 602 606 607 608 610 611 178 613 614 615 616 617 622 623 347 352 626 463 643 K ₁₁₇₀ 589 590 591 592 595 596 600 601 602 606 607 608 610 611 178 223 616 617 237 622 623 347 352 626 463 468 892 K ₁₁₇₁ 589 590 591 592 595 596 600 601 602 606 607 608 610 163 611 178 223 616 617 237 622 623 347 352 463 465 468 469 402 613 311 78 223 616 617 237 622 K ₁₁₇₂ 589 590 591 592 595 596 600 601 602 606 607 608 610 163 611 178 223 616 617 237 622 347 350 352 463 465 468 469 492 589 590 591 592 595 596 600 601 602 606 607 608 610 163 611 178 223 616 617 237 294 303 622 350 354 358 463 465 468 469 492 589 590 591 592 595 596 600 601 602 606 607 610 163 611 178 223 616 617 237 294 303 502 23 589 590 591 592 595 596 600 601 606 607 610 163 611 178 223 616 617 237 294 303 350 354 358 465 467 468 469 492 K ₁₇₇₆ 631 639 640 641 652 236 653 299 321 341 345 633 353 455 470 667 K ₁₇₇₈ 631 639 640 641 652 236 653 299 321 341 345 633 353 455 470 667 K ₁₇₈₀ 631 639 640 641 652 236 653 299 321 341 345 533 419 431 456 470 477 K ₁₈₈₁ <td< th=""><th colspan="3">The set of maximal cliques C - Night Shift</th></td<>	The set of maximal cliques C - Night Shift		
	<i>K</i> ₁₆₇	589 590 591 592 595 596 600 601 602 606 607 608 610 611 178 613 614 615 616 617 619	
		622 623 352 626	
	K ₁₆₈	589 590 591 592 595 596 600 601 602 606 607 608 610 611 178 613 614 615 616 617 622	
		623 347 352 626 463	
$ \begin{array}{lllllllllllllllllllllllllllllllllll$	<i>K</i> ₁₆₉	589 590 591 592 595 596 600 601 602 606 607 608 610 611 178 614 616 617 622 623 347	
		352 626 463 468	
$ \begin{array}{lllllllllllllllllllllllllllllllllll$	<i>K</i> ₁₇₀	589 590 591 592 595 596 600 601 602 606 607 608 610 611 178 223 616 617 237 622 623	
		347 352 626 463 468 492	
	<i>K</i> ₁₇₁	589 590 591 592 595 596 600 601 602 606 607 608 610 163 611 178 223 616 617 237 622	
		623 347 352 463 465 468 469 492	
$ \begin{array}{c} 347\ 350\ 352\ 463\ 465\ 468\ 469\ 492 \\ \\ K_{173} \\ 589\ 590\ 591\ 592\ 595\ 596\ 600\ 601\ 602\ 606\ 607\ 608\ 610\ 163\ 611\ 178\ 223\ 616\ 617\ 237\ 294 \\ 303\ 622\ 350\ 352\ 354\ 463\ 465\ 468\ 469\ 492 \\ \\ K_{174} \\ 589\ 590\ 591\ 592\ 595\ 596\ 600\ 601\ 606\ 607\ 610\ 163\ 611\ 178\ 223\ 616\ 617\ 237\ 294\ 303\ 622 \\ \\ 350\ 354\ 358\ 463\ 465\ 468\ 469\ 492 \\ \\ K_{175} \\ \hline 589\ 591\ 595\ 596\ 600\ 601\ 606\ 607\ 610\ 163\ 611\ 178\ 223\ 616\ 617\ 237\ 294\ 303\ 350\ 354\ 358 \\ \\ 465\ 467\ 468\ 469\ 492 \\ \\ \\ K_{176} \\ \hline 631\ 639\ 640\ 641\ 643\ 648\ 652\ 653\ 657\ 659\ 660\ 661\ 662\ 663\ 366\ 667 \\ \\ \\ K_{177} \\ \hline 631\ 639\ 640\ 641\ 652\ 653\ 657\ 659\ 660\ 661\ 662\ 663\ 353\ 455\ 647 \\ \\ \\ \\ K_{178} \\ \hline 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 201\ 321\ 41\ 345\ 663\ 353\ 455\ 470\ 667 \\ \\ \\ \\ \\ \\ \\ K_{180} \\ \hline 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 321\ 341\ 345\ 533\ 419\ 456\ 470\ 667 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	<i>K</i> ₁₇₂	589 590 591 592 595 596 600 601 602 606 607 608 610 163 611 178 223 616 617 237 622	
		347 350 352 463 465 468 469 492	
$\begin{array}{c} 303\ 622\ 350\ 352\ 354\ 463\ 465\ 468\ 469\ 492 \\ \\ K_{174} \\ 589\ 590\ 591\ 592\ 595\ 596\ 600\ 601\ 606\ 607\ 610\ 163\ 611\ 178\ 223\ 616\ 617\ 237\ 294\ 303\ 622 \\ 350\ 354\ 358\ 463\ 465\ 468\ 469\ 492 \\ \\ K_{175} \\ 589\ 591\ 595\ 596\ 600\ 601\ 606\ 607\ 610\ 163\ 611\ 178\ 223\ 616\ 617\ 237\ 294\ 303\ 350\ 354\ 358 \\ 465\ 467\ 468\ 469\ 492 \\ \\ \\ K_{176} \\ \hline 631\ 639\ 640\ 641\ 643\ 646\ 648\ 652\ 653\ 657\ 659\ 660\ 661\ 662\ 663\ 666\ 667 \\ \\ K_{177} \\ \hline 631\ 639\ 640\ 641\ 642\ 648\ 652\ 653\ 657\ 659\ 660\ 661\ 662\ 663\ 666\ 455\ 667 \\ \\ \\ K_{178} \\ \hline 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 660\ 321\ 661\ 353\ 455\ 470\ 667 \\ \\ \\ \\ K_{179} \\ \hline 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 321\ 341\ 345\ 663\ 353\ 455\ 470\ 667 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	<i>K</i> ₁₇₃	589 590 591 592 595 596 600 601 602 606 607 608 610 163 611 178 223 616 617 237 294	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$		303 622 350 352 354 463 465 468 469 492	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	<i>K</i> ₁₇₄	589 590 591 592 595 596 600 601 606 607 610 163 611 178 223 616 617 237 294 303 622	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$		350 354 358 463 465 468 469 492	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	<i>K</i> ₁₇₅	589 591 595 596 600 601 606 607 610 163 611 178 223 616 617 237 294 303 350 354 358	
$ \begin{array}{rrrr} K_{176} & 631 639 640 641 643 646 648 652 653 657 659 660 661 662 663 666 667 \\ \hline K_{177} & 631 639 640 641 652 653 657 659 660 661 663 353 455 470 667 \\ \hline K_{178} & 631 639 640 641 652 236 653 299 660 321 661 345 663 353 455 470 667 \\ \hline K_{180} & 631 639 640 641 652 236 653 299 321 341 345 663 353 456 470 667 \\ \hline K_{181} & 631 639 640 641 652 236 653 299 321 341 345 353 419 456 470 667 \\ \hline K_{182} & 631 639 640 641 652 236 653 299 321 341 345 353 419 456 470 667 \\ \hline K_{183} & 639 640 641 652 236 653 299 321 340 341 345 353 419 431 456 470 477 \\ \hline K_{184} & 670 671 672 674 682 683 684 145 687 688 195 200 689 691 435 694 454 \\ \hline K_{185} & 670 671 672 674 682 683 684 145 687 688 195 200 225 689 281 435 694 454 \\ \hline K_{186} & 670 671 672 674 682 683 684 145 687 688 195 200 225 689 281 435 694 454 \\ \hline K_{186} & 670 671 672 674 682 683 684 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{187} & 670 671 672 674 682 683 684 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{188} & 671 672 674 682 683 684 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{190} & 671 682 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{190} & 698 699 701 702 707 708 712 717 720 722 723 724 725 726 337 727 728 730 731 732 733 734 \\ \hline K_{193} & 698 699 701 702 707 708 712 213 216 720 722 724 726 337 728 734 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 724 726 337 728 416 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 737 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 37 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 37 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 737 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701$		465 467 468 469 492	
$ \begin{array}{rrrr} K_{177} & 631 639 640 641 646 648 652 653 657 659 660 661 662 663 666 455 667 \\ \hline K_{178} & 631 639 640 641 652 653 657 659 660 661 663 353 455 470 667 \\ \hline K_{179} & 631 639 640 641 652 236 653 299 660 321 661 345 663 353 455 470 667 \\ \hline K_{180} & 631 639 640 641 652 236 653 299 321 341 345 663 353 456 470 667 \\ \hline K_{181} & 631 639 640 641 652 236 653 299 321 341 345 353 419 456 470 667 \\ \hline K_{182} & 631 639 640 641 652 236 653 299 321 340 341 345 353 419 431 456 470 477 \\ \hline K_{183} & 639 640 641 652 236 653 340 341 419 431 477 480 \\ \hline K_{184} & 670 671 672 674 682 683 684 145 687 688 195 200 689 691 435 694 454 \\ \hline K_{185} & 670 671 672 674 682 683 684 145 687 688 195 200 225 689 281 435 694 454 \\ \hline K_{186} & 670 671 672 674 682 683 684 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{187} & 670 671 672 674 682 683 684 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{188} & 671 672 674 682 683 684 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{189} & 671 672 674 682 683 684 145 687 688 195 200 225 689 343 346 351 483 \\ \hline K_{190} & 671 682 145 687 688 195 200 225 689 343 346 351 483 490 \\ \hline K_{191} & 698 699 701 702 707 708 712 717 720 722 723 724 725 726 337 727 728 730 731 732 733 734 \\ \hline K_{193} & 698 699 701 702 707 708 712 213 216 720 722 724 726 337 728 734 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 724 726 337 728 416 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 737 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 737 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 737 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 737 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 737 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 708 712 213 216 720 722 337 416 473 \\ \hline K_{194} & 698 699 701 702 707 7$	<i>K</i> ₁₇₆	631 639 640 641 643 646 648 652 653 657 659 660 661 662 663 666 667	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	<i>K</i> ₁₇₇	631 639 640 641 646 648 652 653 657 659 660 661 662 663 666 455 667	
$ \begin{array}{rrrr} K_{179} & 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 660\ 321\ 661\ 345\ 663\ 353\ 455\ 470\ 667 \\ \hline K_{180} & 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 321\ 341\ 345\ 663\ 353\ 456\ 470\ 667 \\ \hline K_{181} & 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 321\ 341\ 345\ 353\ 419\ 456\ 470\ 667 \\ \hline K_{182} & 631\ 639\ 640\ 641\ 652\ 236\ 653\ 299\ 321\ 340\ 341\ 345\ 353\ 419\ 456\ 470\ 667 \\ \hline K_{183} & 639\ 640\ 641\ 652\ 236\ 653\ 299\ 321\ 340\ 341\ 345\ 353\ 419\ 456\ 470\ 477 \\ \hline K_{183} & 639\ 640\ 641\ 652\ 236\ 653\ 299\ 321\ 340\ 341\ 345\ 353\ 419\ 456\ 470\ 477 \\ \hline K_{183} & 670\ 671\ 672\ 674\ 682\ 683\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 281\ 435\ 694\ 454 \\ \hline K_{186} & 670\ 671\ 672\ 674\ 682\ 683\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 281\ 483 \\ \hline K_{186} & 670\ 671\ 672\ 674\ 682\ 683\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483 \\ \hline K_{186} & 670\ 671\ 672\ 674\ 682\ 683\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483 \\ \hline K_{186} & 671\ 672\ 674\ 682\ 683\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483 \\ \hline K_{186} & 671\ 674\ 682\ 683\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483 \\ \hline K_{186} & 671\ 674\ 682\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483 \\ \hline K_{189} & 671\ 674\ 682\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483\ 490 \\ \hline K_{190} & 671\ 682\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483\ 490 \\ \hline K_{190} & 671\ 682\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483\ 490 \\ \hline K_{190} & 698\ 699\ 701\ 702\ 707\ 708\ 712\ 717\ 720\ 722\ 724\ 726\ 337\ 728\ 734 \\ \hline K_{193} & 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 416 \\ \hline K_{194} & 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 416 \\ \hline K_{194} & 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 416 \\ \hline K_{194} & 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 416 \\ \hline K_{194} & 676$ 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 72	<i>K</i> ₁₇₈	631 639 640 641 652 653 657 659 660 661 663 353 455 470 667	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	K_{179}	631 639 640 641 652 236 653 299 660 321 661 345 663 353 455 470 667	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	K_{180}	631 639 640 641 652 236 653 299 321 341 345 663 353 456 470 667	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	<i>K</i> ₁₈₁	631 639 640 641 652 236 653 299 321 341 345 353 419 456 470 667	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	K_{182}	631 639 640 641 652 236 653 299 321 340 341 345 353 419 431 456 470 477	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	<i>K</i> ₁₈₃	639 640 641 652 236 653 340 341 419 431 477 480	
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	K_{184}	670 671 672 674 682 683 684 145 687 688 195 200 689 691 435 694 454	
$ \begin{array}{c} K_{186} \\ K_{187} \\ K_{187} \\ K_{187} \\ K_{187} \\ K_{187} \\ K_{1670} \\ K_{17} \\ K_{17} \\ K_{188} \\ K_{17} \\ K_{190} \\ K_{17} \\ K_{190} \\ K_{17} \\ K_{191} \\ K_{191} \\ K_{191} \\ K_{192} \\ K_{192} \\ K_{192} \\ K_{193} \\ K_{193} \\ K_{193} \\ K_{194} \\ $	K_{185}	670 671 672 674 682 683 684 145 687 688 195 200 225 689 281 435 694 454	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	K ₁₈₆	670 671 672 674 682 683 684 145 687 688 195 200 225 689 281 483	
$ \begin{array}{c} K_{188} \\ K_{189} \\ K_{189} \\ \hline 671\ 674\ 682\ 684\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483\ 490 \\ K_{190} \\ \hline 671\ 682\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 483\ 490 \\ \hline K_{191} \\ \hline 698\ 699\ 701\ 702\ 707\ 708\ 712\ 717\ 720\ 722\ 723\ 724\ 725\ 726\ 337\ 727\ 728\ 730\ 731\ 732\ 733 \\ \hline 734 \\ K_{192} \\ \hline 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 734 \\ \hline K_{193} \\ \hline 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 416 \\ \hline K_{194} \\ \hline 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 337\ 416\ 473 \\ \hline \end{array} $	Λ ₁₈₇ V	070 071 072 074 082 083 084 145 087 088 195 200 225 089 545 485	
$ \begin{array}{c} K_{189} \\ K_{190} \end{array} = \begin{array}{c} 671\ 674\ 682\ 684\ 143\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 453\ 483\ 490 \end{array} \\ \begin{array}{c} K_{190} \\ K_{191} \end{array} = \begin{array}{c} 671\ 682\ 145\ 687\ 688\ 195\ 200\ 225\ 689\ 343\ 346\ 351\ 453\ 483\ 490\ 494 \end{array} \\ \begin{array}{c} K_{191} \\ K_{192} \end{array} = \begin{array}{c} 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 734 \end{array} \\ \begin{array}{c} K_{193} \\ K_{194} \end{array} = \begin{array}{c} 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 724\ 726\ 337\ 728\ 416 \end{array} \\ \begin{array}{c} K_{194} \end{array} = \begin{array}{c} 698\ 699\ 701\ 702\ 707\ 708\ 712\ 213\ 216\ 720\ 722\ 337\ 416\ 473 \end{array} \\ \end{array}$	κ_{188}	0/1 0/2 0/4 082 083 084 145 08/ 088 195 200 225 089 545 540 551 485	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	Λ ₁₈₉ V	0/1 0/4 002 004 145 00/ 000 195 200 225 009 545 540 551 465 490 671 692 145 687 688 105 200 225 680 242 246 251 452 482 400 404	
K191 698 699 701 702 707 708 712 717 720 722 724 726 337 727 728 730 731 732 733 734 K192 698 699 701 702 707 708 712 213 216 720 722 724 726 337 728 734 K193 698 699 701 702 707 708 712 213 216 720 722 724 726 337 728 416 K194 698 699 701 702 707 708 712 213 216 720 722 337 416 473	К ₁₉₀ К	071 002 143 007 000 173 200 223 007 343 340 331 433 403 470 474 608 600 701 702 707 708 712 717 720 722 723 724 725 726 227 727 728 720 721 722 722	
K_{192} 698699701702707708712213216720722724726337728734 K_{193} 698699701702707708712213216720722724726337728416 K_{194} 698699701702707708712213216720722337416473	N 191	734	
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$\begin{array}{c} K_{193} \\ K_{194} \end{array} \begin{array}{c} 698 \ 699 \ 701 \ 702 \ 707 \ 708 \ 712 \ 213 \ 216 \ 720 \ 722 \ 337 \ 416 \ 473 \end{array}$	K192 K102	698 699 701 702 707 708 712 213 216 720 722 724 726 337 728 416	
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