

Testing the Adaptive Market Hypothesis:

Evidence of time-varying efficiency in emerging foreign exchange markets

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Abstract

This study evaluates the existence of the Adaptive Market Hypothesis (AMH) as an evolutionary alternative to the Efficient Market Hypothesis (EMH) in which the degree of market efficiency is related to market conditions. The Global Financial Crisis is analyzed in order to know if this event has changed the market efficiency of BRICS forex markets. To determine the changes in the degree of efficiency or inefficiency over time, the whole period for each exchange rate has been divided in three different subperiods: pre-crisis, crisis and post-crisis. The data span of daily returns is from 2000 to 2018. We apply and compare the performances of the Adjusted Rescaled Range Analysis (R/S_{adj}), Detrended Fluctuation Analysis (DFA) and Detrending Moving Average Analysis (DMA) through the estimated Hurst exponents. Furthermore, a strict statistical test in the spirit of bootstrapping is conducted to validate the statistical significance of the Hurst exponents. The empirical findings show that BRL/USD, RUB/USD, INR/USD and CNY/USD have been in weak form inefficient when the whole series is considered. Besides, when the whole series is divided only the RUB/USD, CNY/USD and INR/USD support the AMH, shedding light on the dynamics of market efficiency.

Keywords: Adaptive market hypothesis, BRICS forex markets, Hurst exponent, financial crisis, efficiency.

Foreword

This thesis is written as completion of the Master of Science in Economics, at Ghent University. The major of the master is on Financial Institutions and Markets, for that reason I decide to further investigate this interesting and challenging topic, the Adaptive Market Hypothesis.

This thesis would not have been possible without the help of some people. Firstly, I would like to thank Prof. Dr. Michael Frömmel for giving me the opportunity to write this dissertation and his valuable feedback.

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List of used abbreviations

AMH – Adaptive Markets Hypothesis

AUD – Australian Dollar

BRL – Brazilian Real

BCB – Brazilian Central Bank

BDS – Brock, Dechert and Scheinkman

CNY – Chinese Yuan

CAD – Canadian Dollar

CHF – Swiss Franc

DEM – Deutsche Mark

DFA – Detrended Fluctuation Analysis

DMA – Detrending Moving Average Analysis

EMH – Efficient Market Hypothesis

EUR – Euro

Forex – Foreign Exchange

FRF – French Franc

GS – Generalized Spectral

GBP – Great British Pound

IMF – International Monetary Fund

INR – Indian Rupee

IRR – Iranian Rial

JPY – Japanese Yen

MDH – Martingale Difference Hypothesis

RWH – Random Walk Hypothesis

R/S – Rescaled Range

RUB – Russian Ruble

USD – US Dollar

VR – Variance Ratio

WBAVR – Wild Bootstrap Automatic Variance Ratio

ZAR – South African Rand

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1. Introduction

Foreign exchange market efficiency also known as Informational Efficiency (Hallwood & MacDonald, 1994) has been a source of discussion in the international finance sphere. The efficiency or inefficiency behavior in the dynamics of BRICS exchange rates has been not only of special interest for policymakers, monetary and fiscal authorities, but also for investors and traders. The former acts to correct the inefficiencies in order to ensure the optimal functioning of the forex market, reducing the associated risks and ensuring economic stabilization while the latter attempts to exploit the market inefficiencies to make a profit.

According to Fama (1970), the Efficient Market Hypothesis (EMH) has three states of efficiency: weak, semi-strong and strong, which reflect different degrees of information. The weak form explains that current prices reflect all past public feasible information. Semi-strong shows that prices will instantly adjust to the new public information about economic fundamentals and strong form considers that the prices already reflect even the sensitive information. Hence, the weak form market efficiency is a prerequisite for higher types of efficiency. The rejection of it means the rejection of the semi-strong and strong forms (Campbell et al., 1997).

The forex market is characterized as efficient if it fully and instantaneously reflects all the relevant information available to currency traders. Therefore, no profitable information about future exchange rate movements could be obtained by taking into account the past exchange rates, so it means weak-form efficiency (Fama, 1984). Adversely, if the forex market is inefficient, currencies are often incorrectly priced.

The literature of emerging forex markets has been not only centered in testing the traditional EMH but has also allowed the analysis of interesting theoretical frameworks as the Adaptive Markets Hypothesis (AMH) developed in the recent years by Lo (2004, 2005, 2012, 2017) who view that the market efficiency could evolve over time instead of a state that is constant at all times. There are several studies proving the AMH in stock markets from developed and developing countries (Urquhart & McGroarty, 2014; Shi et al., 2016; Ghazani & Araghi, 2014; Ito & Sugiyama, 2009; Boya, 2019), but the research focus on forex market has not been so extensive.

This thesis aims at examining the AMH based on daily returns of BRICS forex markets from 2000 to 2018 with respect to the Global financial crisis. For this purpose, we test the presence of long memory in the market returns in three periods: pre-crisis, crisis and post crisis. This specific event will determine whether the AMH is adequate to explain the time-varying efficiency in the mentioned forex markets based on the Hurst exponents obtained by three approaches: R/S_{dl} , DFA and DMA. Furthermore, a strict statistical test in the spirit of bootstrapping is performed to validate if the Hurst exponent of each of the five exchange rate returns differs from the Hurst exponents obtained from the shuffled series. Accordingly, the whole series and subseries will be classified between weak form efficient or inefficient.

The results obtained are mixed and controversial. The reasons could be the different methods used, the number of observations, the data aggregation and the division of the sample in three sub-periods with different length. For this reason, the comparison of the results is derived from DFA approach due to the mentioned technique is more suitable in short and intermediate time series (Xu et al., 2005). As a result, the efficiency only in the RUB/USD, CNY/USD and INR/USD market is a time-varying phenomenon influenced in this case by the Global financial crisis which adheres to the fundamental notion of the AMH. In addition, the inefficiency in the returns of the BRICS exchange rates are confirmed when the whole series are taking in consideration except for ZAR/USD.

The structure of this thesis is organized as follows. In section 2, the assumptions of AMH are presented and discussed. Some empirical studies and methods are mentioned for developed and developing forex markets based on time-varying efficiency factors. Section 3 shows the forex market in BRICS economies. Section 4 depicts the data set and presents the summary statistics. Section 5 discusses the followed methodology. In section 6, the results of the empirical study are reported along with some discussion. Finally, some general conclusions are provided in section 7.

2. Adaptive Market Hypothesis

2.1 Assumptions

The AMH is based on three main assumptions:

(i) Individuals are neither always rational nor irrational, they act in their own self-interest, make mistakes, learn and adapt to different environments. These changes in the environment could happen due to a change in the composition of the population¹ (Figure 2.1). As a consequence, market inefficiencies do exist. Menkhoff (1998) differentiates between “rational arbitrageurs” and “less rational noise traders”. The former as traders who behave rationally, primarily relying on the fundamental analysis, so they keep the exchange rate in line with macroeconomic fundamentals and help to stabilize markets around a new equilibrium. The later as some traders that primarily relying on “sentiments” and/or “noises” so they are destabilizing speculators.

(ii) Financial market dynamics are driven by our interactions or evolution. From an evolutionary perspective, it means that profit opportunities do exist occasionally in the financial markets, they are influenced by changes in business conditions, number of competitors, composition of investors. Shifts in these factors over time are suitable to affect the relationship between risk and reward. As a result, market equilibrium is neither guaranteed nor likely to occur at any point of time.

(iii) Survival is the primary objective of any market participant based on innovation and the ability to adapt to market changes. The AMH recognizes the existence of different forms of market dynamics: trends, cycles, bubbles, and crashes. Each of these forms requires different investment strategies. Trading strategies are introduced, mutate to survive, or face abandonment (Soufian et al., 2014). In consequence, investment strategies may perform well only in certain environments.

Therefore, Lo (2004, 2005) suggests the AMH to be considered as an addition to the EMH that explains different anomalies, and not as a deny or replacement of it. His hypothesis reconciles the EMH with the notion of “bounded rationality”² explained by behavioral economics through an evolutionary approach to economic interactions.

Nevertheless, there is a main point which differentiate the AMH explained by Lo (2004, 2005, 2012, 2017) and the EMH explained by Fama (1984). The EMH assumes that market is frictionless, this means that

¹ Population is referred as a group of market participants that behaves in a similar way, such as private investors, mutual funds, hedge funds, banks, etc.

² Bounded rationality is “the premise that individuals are influenced by their tastes, values, past judgments, and limits of their cognitive process, resulting in a satisfactory outcome” (Baker et al., 2017).

return predictability is not possible. So, prices in an efficient market should follow a random walk (Levich, 1985) or the less restrictive martingale process³ (Campbell et al., 1997). Several studies observe random walk or martingale behavior in forex returns (Belaire-Franch & Opong, 2005; Rashid, 2006; Yang et al., 2008; Chiang et al., 2010; Al-Khazali et al., 2012; Salisu et al., 2016). However, the literature has not reached to a unique conclusion about the validation of weak-form efficiency. This mixed of evidence could be caused by different degree of markets development (Levich, 2001), the method used to test market efficiency (Rossi, 2013) and the covered period (Lazăr et al., 2012). Additionally, the EMH affirms that the market is always weakly efficient, even during crises.

Conversely, AMH proposes market frictions and asserts that markets evolve over a period, therefore the return predictability may arise time to time based on central banks interventions, the exchange rate regime, the divergence between the equilibrium and official rate due to a parallel black market, the exposure to crisis, among others (Charles et al., 2012). These events are associated with a high degree of uncertainty and have strong implications for the psychology of market participants, in the way of how they incorporate new information to prices (Kim et al., 2011).

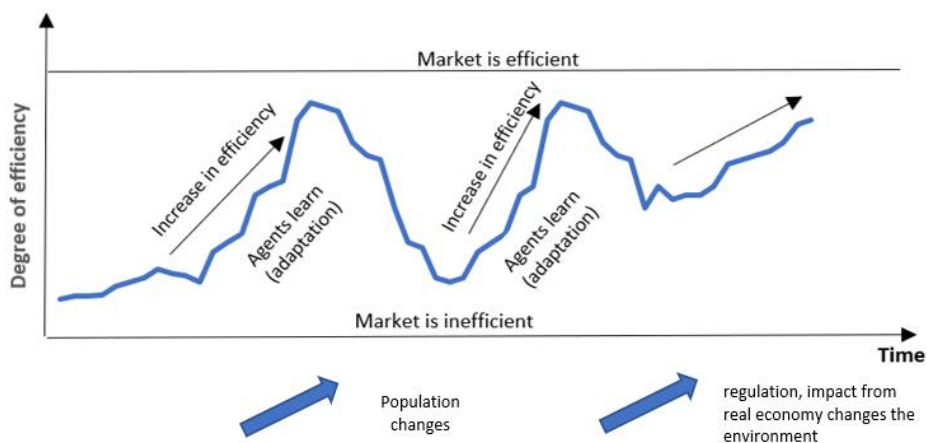


Figure 2.1 AMH and Market Efficiency⁴

³ The main difference between the random walk and the martingale model is that the martingale model does not require Independent and identically distributed (IID) increments so this feature allows for time-varying volatilities. A random walk is more restrictive than a martingale.

⁴ This graph was extracted from the book written by Frömmel, (2017). *Finance 2: Asset Allocation and Market Efficiency*, second edition.

2.2 Time-varying efficiency factors in developed and developing forex markets

With regard to developed forex market, Neely et al. (2009) analyze the AMH through technical trading rules for European exchange rates from 1973 to 2005. The authors show that both institutional and behavioral factors might have delayed the implementation of effective investment strategies. While, Charles et al. (2012) examine the return predictability of five major forex rates, AUD, GBP, CAD, JPY and CHF relative to the USD, from 1975 to 2009 using daily and weekly nominal exchange rates based on MDH which include the WBAVR by Kim (2009), GS test by Escanciano & Velasco (2006), and Domínguez & Lobato (2003) consistent tests. They find evidence that the returns of the exchange rate are unpredictable in most of the tested periods, however, the short-term inefficiencies episodes of return predictability are attributed to changes in market conditions by events such as central bank interventions and financial crisis. Hence, they prove that markets can be both efficient and inefficient over a sample period as efficiency is measured at each time interval. Indeed, these findings are consistent with the AMH. In addition, Almail & Almudhaf (2017) examine the efficiency of GBP, the sample covers the period from January 1779 to April 2016. The authors use the Automatic VR and Automatic Portmanteau tests, they find evidence that currency fluctuates and shifts between independent and dependent return predictability during the sub-periods of the last three centuries attributed to episodes of irrationality, shocks to markets, several behavioral and fundamental factors.

Another strand of literature focuses on the emerging forex markets. For instance, Da Silva et al. (2007) apply Hurst exponents to test the efficiency of the Brazilian forex market relative to USD from 1995 to 2006 using daily data. Based on the Hurst exponent, the authors conclude that Brazilian forex market is inefficient before the currency crisis in 1999, but after the crisis, it tends to efficiency. The same forex market is analyzed by Stošić et al. (2015) who indicate an increase in market efficiency in the transition of the regime of the exchange rate that occurs in 1999 based on multifractal DFA. While Lazăr et al. (2012) examine the performance of daily euro forex rates for RUB through MDH based on GS test by Escanciano & Velasco (2006) from January 2004 to February 2011. The results show that during the crisis higher predictability is detected for the Russian forex markets.

Furthermore, a series of recent studies have examined the evolution of the market efficiency for the Indian exchange rate. For example, Khuntia et al. (2018) and Kumar (2018), who have used an identical data sample of daily observations from 1999 to 2017. The formers based on their analysis in WBAVR test for linear dependence and GS tests for nonlinear dependence in forex returns. They confirm the time-varying efficiency can be linked to the existence of crisis, market reforms, and political changes.

While Kumar (2018) compare the evolution of market efficiency relative to USD, GBP, EUR and JPY by non-overlapping and overlapping moving window analysis based on the Automatic VR test and Belaire-Franch and Contreras (2004) rank based tests. The results provide evidence about the unpredictable behavior of the Indian exchange rates for most of the times, however, around major macroeconomic events, there are episodes of the inefficient behavior for the Indian exchange rate. These results of inefficiency during the financial crisis are reaffirmed by Kumar (2016) who gives special attention to the behavior of returns of INR before and after the 2008 financial crisis. The author mentions that after 2008, the effects disappear for all currencies (EUR, USD, GBP, JPY) which indicate that the markets become more efficient. Additionally, he emphasizes that this result can be also associated to the increased intervention in the forex markets after the crisis by the Reserve Bank of India.

Besides, Salisu et al. (2016) study the forex markets in nine Asia Pacific countries including China. The forex markets of all the selected countries are efficient under the full sample period based on MDH analyzed by WBAVR (Kim, 2009) and GS test by Escanciano & Velasco (2006). The authors find that forex market efficiency could be inconsistent over time due to changes in policies and events. Moreover, the impact of the Asian financial crisis on the financial markets is studied by Jeon & Seo (2003). The authors show weak market efficiency immediately after the crisis than before the crisis. The market efficiency is recovered quickly, as shown by the retaining cointegrating relationships for the pairs of the spot-forward exchange rates. Ning et al. (2017) also examine the forex market in China using multifractal DFA of the onshore and offshore RMB/USD spot exchange rate series. They conclude that market efficiency is associated with changes in intervention of the People's Bank of China.

Yaya et al. (2019) examine the level of exchange rates market efficiency and volatility at three market phases of USD/ZAR based on daily data from 2 January 1995 to 8 June 2016. They apply the modified Hurst exponent which shows that the exchange rate has been in weak form of efficiency since 2001. In this case, the theory of AMH could not be used to explain the behavior of the returns in USD/ZAR forex series. Conversely, Olufemi et al. (2017) examine the efficiency of 10 forex markets in sub-Saharan Africa in presence of structural break through WBAVR (Kim, 2009) in which the results show that only Burundi and Ghana forex markets are efficient before and after the structural break. While, Gambia and Madagascar are not efficient. The conclusion of this study is different when the efficiency is examined by BDS independence test which reports that all the forex markets are not efficient, supporting the AMH that market efficiency tends to vary with time.

Furthermore, Abounoori et al. (2012) show that the Iranian forex market (IRR/USD) is weak form inefficient over the whole period 2005 - 2010 but also the degree of inefficiency is not constant when they split the data in various subperiods through DFA technique. The relationship between return predictability and market conditions in the Ugandan forex market has been investigated by Katusiime et al. (2015) for the period from January 1994 to June 2012 based on VR test. They find that forex market's efficiency is dependent on market conditions. Gupta et al. (2018) analyze the directional predictability using the quantilogram proposed by Linton & Whang (2007) for BRICS forex markets. The authors use monthly data from 1812 until 2018, respectively for the dollar-based exchange rates of the BRICS countries. They evidence that predictability holds at certain parts of the unconditional distribution of exchange rate returns. This finding supports the AMH indicating that return predictability may arise from time to time.

Finally, Yamani (2017) indicates that both developed and developing forex markets are mostly efficient during periods of low volatility. Conversely, when the market conditions deteriorate, only developing markets tend to be generally inefficient, evidenced by both the rejection of the RWH and the profitability of technical trading rules.

3. Forex Market in BRICS

Although each member of BRICS is quite different expressed by the internal politics and economics from each other. The integration of the BRICS countries into the world economy is one of the major developments in the international economics for more than a decade. The trade among these economies with the rest of the world has grown at fast speed. This strong economic performance has been associated with the high level of foreign direct investment in the private sector (Ruzima & Boachie, 2018). In 2018 the combined economy of the BRICS made up 23,5% of the global GDP (IMF, 2018). Furthermore, the contribution from this bloc to global output is expected to overcome the G7 countries by 2050 (Wilson & Purushothaman, 2003).

The financial markets in the BRICS economies have similarly expanded in an accelerated manner. The movements of these emerging bloc currencies, BRL, RUB, INR, CNY, and ZAR into the global financial markets have turned these economies into an important destination for global capital. Moreover, the BRICS currencies consistently have been among the world's twenty most heavily traded currencies. In the last triennial survey 2016 of forex market conducted by the Bank for International Settlements, the forex turnover of BRICS currencies was around 8%, being the CNY currency the leader of the group despite of the fact that China began promoting its currency internationalization after the other BRICS countries (Russian Institute for Strategic Studies, 2017). However, there are few studies that examined the forex markets of BRICS countries as a block (Aloui et al., 2011).

From one point of view, one of the major determinants of the forex market efficiency is the exchange rate regimes applied (Azad, 2009). Even if the probability of change across regimes happens infrequently, it generates an increasing source of uncertainty that investors want to hedge against (Ang & Timmermann, 2012). All of BRICS's forex markets have experienced transitions from regimes caused by economic, political, and market factors (Ilzetzki et al., 2019). Considering the IMF classification, a floating exchange⁵ rate has been adopted from Brazil since 1999, India since 1993 and South Africa since 1995. While Russia has a free-floating arrangement⁶ since 2015 before it was categorized as other managed arrangement⁷. Furthermore, China has presented a series of changes since 2005 when they maintained an exchange rate

⁵ A floating exchange rate is "largely market determined, without a predictable path for the rate". In floating regime, the forex market intervention could be either direct or indirect and help to control the rate of the change and prevent the excessive fluctuations in the exchange rates (IMF, 2019).

⁶ Free floating "if intervention occurs only exceptionally and aims to address disorderly market conditions" (IMF, 2019).

⁷ Other managed arrangement is characterized by periods in which the volatility of forex market conditions block the use of a clearly definition of exchange rate regime (IMF, 2019).

pegged to the USD. It was reclassified as a crawling peg⁸ until the period of the Global financial crisis. From June 2008 to June 2010, a stabilized arrangement⁹ was conducted. The classification reverted to a crawl-like arrangement until 2015. In 2016, the classification was changed to other managed arrangement. In 2017, China was classified into a stabilized arrangement. While in the last report 2018, China was reclassified into crawl-like arrangement.

In general, the monetary policy objectives and the strategies pursued by the authorities have vary significantly in each BRICS forex market. However, the analysis of time vary efficiency due to different exchange rate regimes will not be analyzed in this present study.

From another point of view, if we compare between BRICS forex markets and developed economies, the former has experienced more pronounced exchange rate volatility than developed economies due to these markets are more sensitive to the effects of the financial crisis than other countries. Thus, the analysis of the impact of financial crisis is important because the crisis are a recurrent phenomenon with an increased risk, affecting the international trade flows, balance of payments and allocation of resources in national and international economy (Horta 2013; Lazăr et al., 2012). These tumultuous movements in forex markets have altered the market conditions triggering the change in the degree of the return predictability (Lo, 2004; Charles et al., 2012; Aroskar et al., 2004). Tsangarides (2012) analyzes the forex rate performances in the emerging markets during the financial crisis and post crisis and asserts the financial channel plays an important role during the crisis. In this sense, Frankel & Saravelos (2012) indicate that the appreciation and depreciation of currencies are significant and show the exchange market pressure during this period. Karmakar et al. (2017) rank the major financial crisis in the new millennium that have impacted BRICS economies, Argentine financial crisis (2001-2002), Global financial crisis (2008-2009), Eurozone crisis (2010) and the Greek financial crisis (2015). But also, we could mention the Russian financial crisis (2014-2016). In this study, we evaluate the efficiency of BRICS forex markets in the context of the Global financial crisis.

⁸ In crawling peg, “the currency is adjusted in small amounts at a fixed rate or in response to changes in selected quantitative indicators” (IMF, 2019).

⁹ The exchange rates in countries with stabilized and crawl-like arrangement are often adjusted as a reaction to external events (IMF, 2019).

4. Data

The data consists of the daily BRICS/ USD exchange rates. We use the USD as exchange rate due to it is the most used currency in financial markets and international trade. The different time series cover the period from January 3, 2000 to December 31, 2018 with a total of 4956 data points for all the currencies except for CNY. The period for CNY is taken from July 25, 2005 to December 31, 2018 with 3506 data points because of the Chinese reform implemented in which they abandoned the fixed exchange rate system (Cao et al. 2018). All the data is extracted from DataStream. The evolution in levels of BRICS's currencies over USD exchange rate is presented in Figure 4.1(A). An upward movement indicates appreciation of the BRICS's currency to the USD, while a downward indicates depreciation.

First, we calculated the logarithmic return of each BRICS/ USD, where $p(t)$ represents the price of each BRICS/USD exchange rate at the time (t):

$$r(t) = \ln p(t) - \ln p(t - 1)$$

Equation 4.1 Daily returns of the exchange rate

The evolution of the returns for exchange rate is illustrated in Figure 4.1(B). The descriptive statistics and unit root tests for the return series are providing in Table 4.1. Starting with the mean of the returns, the BRL, RUB, INR and ZAR present negative returns across the reference currency (USD), the only positive return is for CNY. Furthermore, the most volatile return is for South Africa as indicated by the associated standard deviation while the Chinese returns are the least volatile. The kurtosis values show that the return series is leptokurtic, thus means the tail is heavier than normal. The Jarque Bera test confirms the significant non-normality in all the daily returns for the BRICS exchange rates. Furthermore, we perform the Augmented Dickey-Fuller which rejects the null hypothesis of a unit root in all the exchange return series. Thus, all the return series are stationary at the 1% significance level.

In order to study the effect of the Global financial crisis in the efficiency of BRICS/ USD exchange rates, we have divided the whole sample into three subsamples. The subsample 1 consists of data before July 2, 2007 considering it as the pre-crisis period, while the subsample 2 that is the period of crisis has data from July 2, 2007 to November 30, 2009 and the subsample 3 presents the data after the global financial crisis until December 31, 2018. In the specific case of RUB/USD, we have split the sample additionally taking in consideration the endogenous financial crisis that Russia experimented during the period 2014-2016.

Thus, subsample 2 and subsample 4 are the global financial crisis and the Russian financial crisis, respectively.

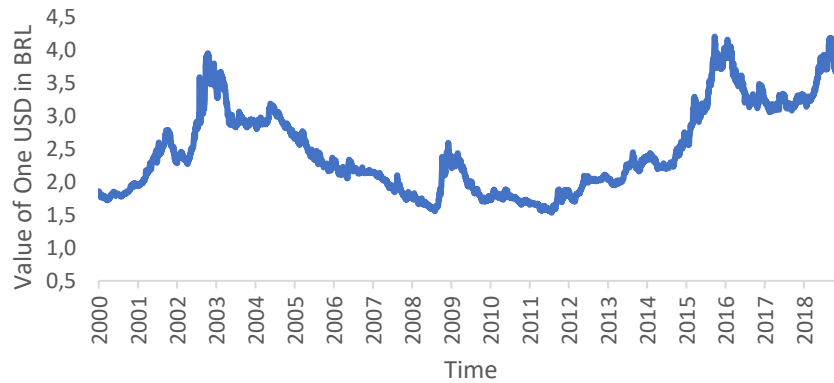
	BRL/USD	RBL/USD	IND/USD	CYN/USD	ZAR/USD
Mean	-0,016	-0,019	-0,010	0,005	-0,017
Maximum	11,778	15,523	3,064	1,150	8,523
Minimum	-9,677	-14,268	-3,251	-1,810	-9,808
Standard Deviation	1,005	0,770	0,378	0,142	1,055
Skewness	-0,118	-0,261	-0,267	-0,541	-0,284
Kurtosis	14,544	72,095	10,456	19,728	7,937
Jarque-Bera ¹	27531,700	985918,900	11537,800	41048,130	5099,600
ADF ^{1,2}	-53,339	-19,255	-28,210	-23,910	-68,479
Observations	4956	4956	4956	3506	4956

Notes:

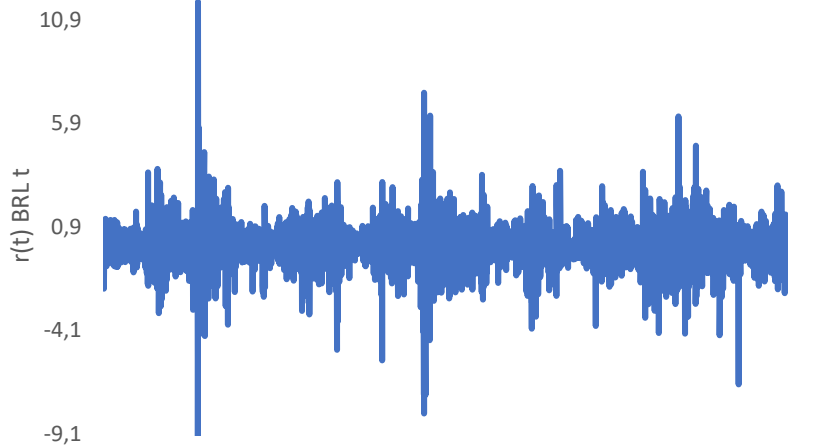
- 1.- Null hypothesis is rejected at the 1% level.
- 2.- ADF is the Augmented Dickey-Fuller unit root tests.

Table 4.1 Descriptive statistics

(A)

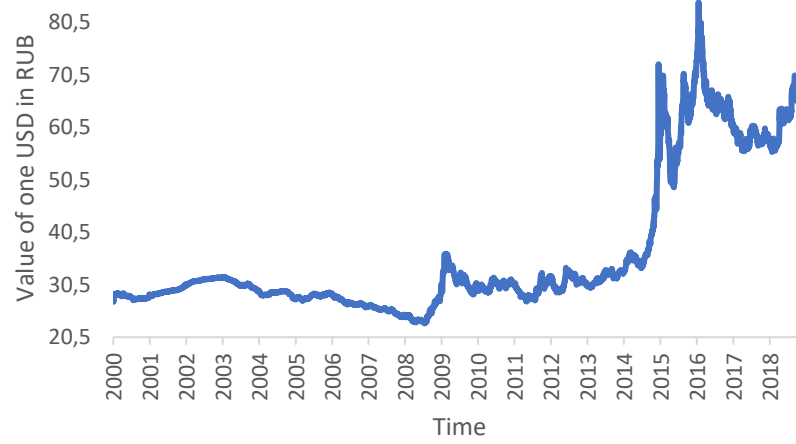


(B)



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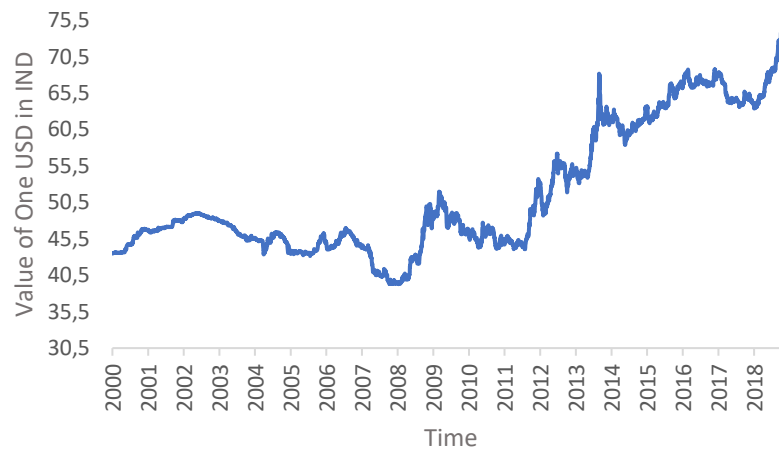
(A)



(B)

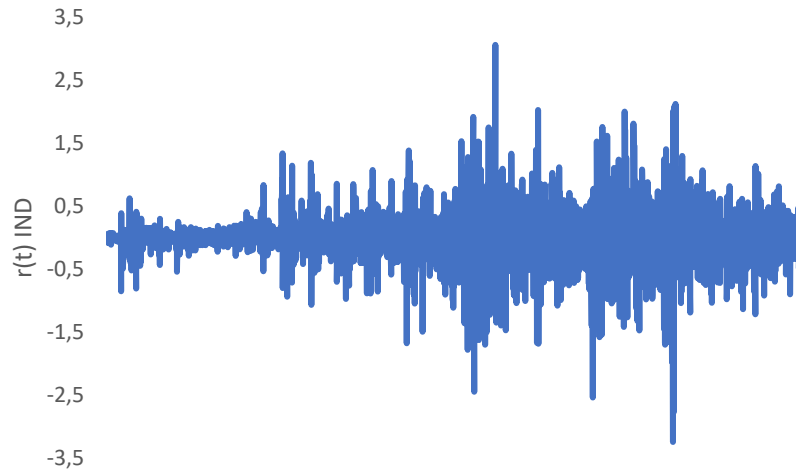


(A)

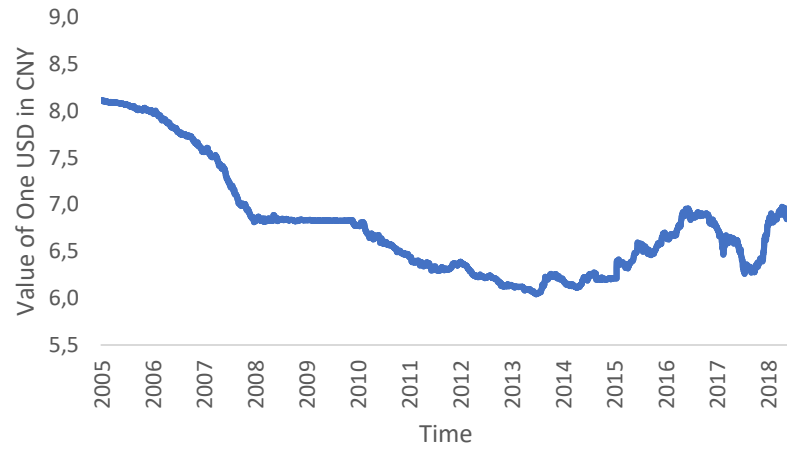


cont.

(B)



(A)

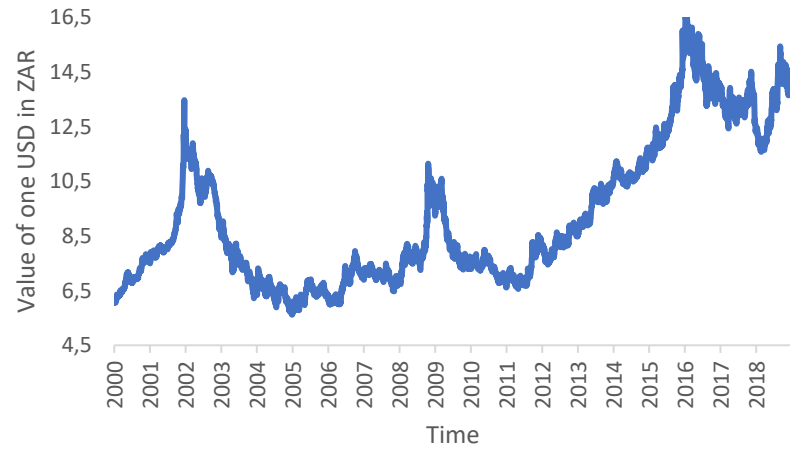


(B)



cont.

(A)



(B)

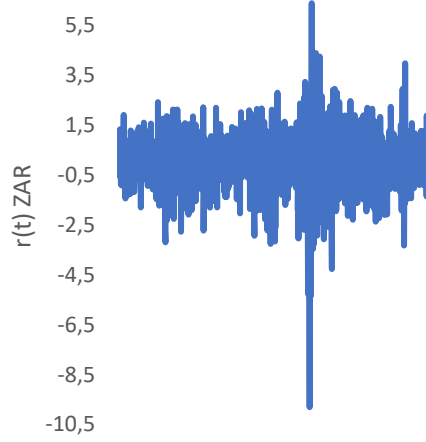


Figure 4.1 Evolution of each BRICS exchange rate (A) in levels and (B) in returns

5. Methodology

In financial literature, the Hurst exponent has been used to study the dynamics and the efficiency of forex markets, stock markets, among others from a long memory point of view. The presence of long memory has been widely used as an evidence against the weak form market efficiency (Tabak & Cajueiro, 2007; Jagric et al., 2005; Onali & Goddard, 2011). There are few studies that have applied Hurst exponents to test the AMH (Hiremath & Narayan, 2016; Hull & McGroarty, 2014). Furthermore, the examination of the degree of efficiency through Hurst exponents have been employed in different circumstances. For instance, Cajueiro et al. (2009), Wang et al. (2009), Yang et al. (2019) and Qin et al. (2015) examine the efficiency in the context of local market reforms or interventions. While Horta et al. (2014), Lahmiri (2015), Ma et al. (2016), Mynhardt et al. (2014) and Rizvi & Arshad. (2016) use the Hurst exponent to evaluate the efficiency in the context of financial crisis. In this study, we will use the Hurst exponent to measure the efficiency of BRICS forex market during the period of the global financial crisis.

There are different approaches to calculate the Hurst exponent. We study and compare the performances of the Adjusted Rescaled Range Analysis (R/S_{adj}), Detrended Fluctuation Analysis (DFA) and Detrending Moving Average Analysis (DMA) in its three forms: Backward Detrending Moving Average (BDMA), Forward Detrending Moving Average (FDMA) and Central Detrending Moving Average (CDMA) to determine the Hurst index (H) for BRICS/ USD exchange rates. It is important to say that there is not an unique agreement about which is the best method to apply and under which parameters (Shao et al., 2012; Bashan et al., 2008).

The H can be interpreted for each method as follows:

- If $H = 0,5$ the time series is independent (no memory of previous values), there is a lack of correlation. Hence, the time series display a random walk behavior (market efficiency).
- If $0,5 < H < 1$ the time series has persistent long-range correlations. So, if the series has been up or down in the last period then the chances are that it will continue to be up or down, respectively (market inefficiency). The presence of long-range correlations shows evidence for futures returns.
- If $0 < H < 0,5$ the time series has antipersistent long-range correlations. This means that whenever the time series have been increasing in the last period, it is more likely that it will be decreasing in the next period (market inefficiency).

5.1 Adjusted Rescaled Range Analysis

This technique was first proposed by Hurst (1951) and popularized by Mandelbrot and Wallis (1969) and Mandelbrot (1972) as a long term dependency analysis of the stock markets. We follow the structure explained by Weron (2002) which distinguish the use of the Adjusted Rescaled Range (R/S_{al}) to improve the performance in very small series. The returns of the time series x_t of length N are divided into s subseries of length n , where n is an integral divisor of N . For each subseries the following procedure must be applied:

1. Find the mean (E_S) and standard deviation (S_S).
2. Normalize the data ($Z_{i,s}$) by subtracting the sample mean of the returns.

$$X_{i,s} = Z_{i,s} - E_S, i = 1, \dots, n$$

Equation 5.1.1 Mean adjusted series

3. Construct the profile ($y_{i,s}$) that is the cumulative summation series as follows:

$$y_{i,s} = \sum_{j=1}^i X_{j,s}, i = 1, \dots, n.$$

Equation 5.1.2 Cumulative time series

4. Find the range R_i .

$$R_i = \max_{1 \leq i \leq n} (y_{i,s}) - \min_{1 \leq i \leq n} (y_{i,s})$$

Equation 5.1.3 Range series

5. Calculate the mean value of the R/S for all subseries s of length n .

$$(R/S)_n = \frac{1}{s} \sum_{i=1}^s \frac{R_i}{S_i}$$

Equation 5.1.4 Mean value of the R/S

6. The R/S statistic asymptotically follows the relation $(R/S)_n \sim cn^H$. The value of H can be obtained by a linear regression over a sample of increasing time horizons.

$$\log ((R/S)_n) = \log c + H \log n$$

Equation 5.1.5 Linear regression in R/S

However, for small values of n there is a significant deviation from the 0,5. Sánchez et al. (2008) suggested that the estimation of modified R/S denoted by R/S_{al} is better than R/S , especially with short series. So theoretical values for R/S statistics are usually approximated by:

$$E(R/S)_n = \begin{cases} \left(\frac{n-\frac{1}{2}}{n}\right) \left(\frac{\Gamma(n-\frac{1}{2})}{\sqrt{\pi}\Gamma(\frac{n}{2})}\right) \left(\sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}\right) & \text{for } n \leq 340, \\ \left(\frac{n-\frac{1}{2}}{n}\right) \left(\frac{1}{\sqrt{\frac{n\pi}{2}}}\right) \left(\sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}\right) & \text{for } n > 340, \end{cases}$$

Equation 5.1.6 Corrected Hurst exponent

where Γ is the Euler gamma function. This equation is a modification of Anis and Lloyd (1976). The term $\left(\frac{n-\frac{1}{2}}{n}\right)$ was added by Peters (1994) to improve the performance for very small n .

7. The Adjusted Rescaled Range statistics (R/S_{al}) is calculated as:

$$(R/S_{al})_n = (R/S)_n - E(R/S)_n + E(R/S_{al})_n,$$

Equation 5.1.7 Adjusted Rescaled Range

$$\text{where } E(R/S_{al})_n = \sqrt{\frac{1}{2}\pi n}.$$

Similarly, the Hurst exponent using R/S_{al} statistics will be the slope obtained by running a simple linear regression with $\log(R/S_{al})_n$ as a dependent variable and $\log n$ as an independent variable.

5.2 Detrended Fluctuation Analysis

We follow the DFA technique proposed by Peng et al. (1995). This technique removes the local trends through the least-square regression fit. The framework is as follow:

1. Construct the profile y_i that is the cumulative summation series as follows:

$$y_i = \sum_{t=1}^i (x_t - \bar{x}), \quad i = 1, 2, \dots, N,$$

Equation 5.2.1 Cumulative summation series

where $\bar{x} = \frac{1}{N} \sum_{t=1}^N x_t$ is the sample mean of the returns.

2. Divide the profile y_i into $N_n = \left\lfloor \frac{N}{n} \right\rfloor$ non-overlapping boxes of equal length n . In some cases, the length N of the series is not a multiple of the time scale n . Therefore, a short part at the end of the profile will remain. In order to consider this part of the series, the same procedure is repeated started from the end. So, $2N_n$ boxes are obtained.

3. In each box, we fit the integrated time series by using a polynomial function. Let \tilde{y}_i be the local trend function for each $2N_n$ boxes:

$$\tilde{y}_i = ai + b,$$

Equation 5.2.2 Local trend function

where a and b are the slope and the intercept of the local trend respectively. The local trend in each box can be obtained by local an ordinary least-squares fit of the data points in that box.

4. We detrend the integrated time series y_i by subtracting the local trend \tilde{y}_i in each box. The residual series ε_i is divided into N_n overlapped boxes with the same size n , where $N_n = \frac{N}{n} - 1$. Each box can be denoted as $\varepsilon_v = \varepsilon(l + i)$ for $1 \leq i \leq n$, respectively, where $l = (v - 1)n$.
5. The local fluctuation function $F_v(n)$ is the v th box is defined as the root mean square of the residuals.

$$F_v^2(n) = \frac{1}{n} \sum_{i=1}^n \varepsilon_v^2(i)$$

Equation 5.2.3 Local fluctuation function

6. The overall fluctuation function $F(n)$ is calculated as the root mean square of the detrended time series as a function of the segments size n .

$$F(n) = \left\{ \frac{1}{N_n} \sum_{v=1}^{N_n} [F_v^2(n)] \right\}^{\frac{1}{2}}$$

Equation 5.2.4 Overall fluctuation function

As the box size n varies in proper scaling ranges, we can determine the power-law relation between the function $F(n)$ and the box size n : $F(n) \sim n^H$, where H signifies DFA scaling exponent or better known as the Hurst exponent.

5.3 Detrending Moving Average Analysis (DMA)

We follow the procedure of DMA initially proposed by Arianos and Carbone (2007). The principal difference between the DFA and DMA algorithms is the determination of the local trend function \tilde{y}_i . The DMA algorithm does not need a division of the series in boxes. The scaling property is obtained by the moving average.

1. Construct the profile y_i that is the cumulative summation series as follows:

$$y_i = \sum_{t=1}^i (x_t - \bar{x}), \quad i = 1, 2, \dots, N,$$

Equation 5.3.1 Cumulative summation series

where $\bar{x} = \frac{1}{N} \sum_{t=1}^N x_t$ is the sample mean of the returns.

2. Calculate the moving average function \tilde{y}_i in a moving window, where n is the window size. The n is ranging from 2 to a maximum value n_{max} depending upon the size of the series.

$$\tilde{y}_i = \frac{1}{n} \sum_{k=-[(n-1)\theta]}^{[(n-1)(1-\theta)]} y_{i-k}$$

Equation 5.3.2 Moving average function

The θ is the position parameter with the value varying from $[0, 1]$. The moving average function considers $[(n-1)(1-\theta)]$ data points in the past and $[(n-1)\theta]$ data points in the future.

Three cases have been proposed by (Xu et al., 2005):

- The first case, BDMA, when $\theta = 0$ so the moving average function \tilde{y}_i is calculated over all the past $n - 1$ data points of the signal.
 - The second case, $\theta = 0.5$ corresponds to CDMA, where the moving average function \tilde{y}_i contains half past data points and half future data points in each window.
 - The third case named FDMA, $\theta = 1$, where y_i considers the trend from future $n - 1$ data points.
3. Detrend the series by removing the moving average functions \tilde{y}_i from y_i , and obtain the residual sequence.

$$\varepsilon_i = y_i - \tilde{y}_i,$$

Equation 5.3.3 Residuals

where $n - [(n-1)\theta] \leq i \leq N - [(n-1)\theta]$. The residual series ε_i is divided into N_n non-overlapped boxes with the same size n , where $N_n = \frac{N}{n-1}$.

Each box can be denoted as $\varepsilon_v = \varepsilon(l+i)$ for $1 \leq i \leq n$, respectively, where $l = (v-1)n$.

4. The local fluctuation function $F_v(n)$ is the v th box is defined as the root mean square of the residuals.

$$F_v^2(n) = \frac{1}{n} \sum_{i=1}^n \varepsilon_v^2(i)$$

Equation 5.3.4 Local fluctuation function

5. The overall fluctuation function is calculated as follows:

$$F(n) = \left\{ \frac{1}{N_n} \sum_{v=1}^{N_n} [F_v^2(n)] \right\}^{\frac{1}{2}}$$

Equation 5.3.5 Overall fluctuation function

As the box size n varies in proper scaling ranges, we can determine the power-law relation between the function $F(n)$ and the box size n as $F(n) \sim n^H$, where H signifies DMA scaling exponent or better known as Hurst exponent.

6. Results

To check the sensitivity of our results to the method of calculation, the analysis of the memory behaviors for BRICS daily exchange rates are provided by means of R/S_{al} , DFA and DMA approaches. We have examined the Hurst exponent calculated over the entire period for the BRICS exchange rates, the results are shown in the first column of Table 6.1 - 6.5. Taking account of the whole series for each BRICS's exchange rates and their estimated H , we can find that almost all the exchange rates with the exception of ZAR/USD through the proposed methods are found to be persistence that means that H is higher than 0,5.

Furthermore, the long memory of financial time series could present time-varying features because of the exogenous and endogenous shocks and their significant effects on the dynamics of financial markets (Guangxi et al., 2014). For this reason, this study also tests three subseries which have been split taken into consideration three periods: before the Global financial crisis (sub 1), during the crisis (sub 2) and post crisis (sub 3), respectively. The purpose is to find out whether or not the Global financial crisis can change the efficiency of the forex exchange rate in the defined sub-periods as proof of the theory of AMH. The mentioned theory explains that markets evolve, adapt and switch between efficiency and inefficiency due to events (economic and political crisis, bubbles) or structural changes, causing that the market efficiency varies in degree at different times (Lo, 2005).

Each sub-series is injected into the same analysis procedure as for the whole series. Based on the results of within-countries testing, we find the forex market in each BRICS countries has improved their efficiency. During the global financial crisis, all the markets present larger H than the respective H for the pre-crisis period which means that all forex markets moved towards long memory and persistence during this period. Cajueiro et al., (2009) mentions that the Hurst exponent increases during crisis periods because of the reduction in the investor base and liquidity inducing to a decrease in market efficiency.

After the crisis, the H is more in line to reach the 0,5 showing that most of the emerging markets have recovered in terms of improved market efficiency (Sensoy & Tabak, 2016). Based on this analysis, we could say that efficiency of these forex markets have varied from time to time and the Global financial crisis has led to markets that are more efficient and mature (Vieito et al.,2016), consistent with the implications of the AMH.

However, the estimation of the Hurst exponent H alone is not enough (Weron, 2002). We also need a measure of the significance of the results to avoid mistakes in the interpretation of the long term memory

in Hurst exponents larger than 0,5 (Couillard & Davison, 2005). Therefore, the study conducts an analysis performed on the shuffled series to verify if the original sample presents the same memory behavior as the shuffled series. The shuffled procedure is obtained by generating a random permutation of the array elements of the original time series to extract possible memory behaviors (Jiang et al., 2014). This means that 500 series were built from the original series. The data points of each of the 500 series are the same as the original series but randomly ordered. The randomly chosen realization of the shuffled time series allows us to obtain a distribution of 500 Hurst exponents, from which we calculate the mean, the standard deviation and percentiles 5% and 95%. The shuffled time series is plotted together with the original sample for the whole series of each exchange rate (Fig.6.1- 6.5). In each figure, we can observe the power-law scaling behaviors between the fluctuation function $F(n)$ and the box size n . The lines in each figure are the power-law fits the data points in corresponding scaling ranges, which covers no more than three orders of magnitude.

For the BRICS exchange rates, the shuffled series have a close approach to 0,5, giving the impression that the return of the exchange rates may exhibit different memory features from its shuffled series. However, this deviation of the Hurst exponent from 0,5 does not necessarily imply that the exchange rates are inefficient (Yang et al., 2019). A strict statistical test in the spirit of bootstrapping is performed to identify the memory behaviors in original series and shuffled series more roughly to control the quality of the algorithm applied to finite samples. Jiang et al. (2014) mention that the behaviors present in the series are eliminated when shuffling the series in the bootstrap procedure. This procedure would allow us to determine whether the Hurst exponents are statistically different in the three mentioned periods. The null hypothesis is that the original series has the same memory behavior as the shuffled series, thus: $H = \langle Hs \rangle$, where $\langle Hs \rangle$ is the mean of all Hs values. Thus, we perform a two-tailed p-value, which is described as (Jiang et al., 2014):

$$p = Prob(|Hs - \langle Hs \rangle| > |Hs - \langle Hs \rangle|)$$

Equation 6.1 Two-tailed p-value

For the whole series considering the statistical test: the BRL/USD is found to offers evidence in favor of long-term correlations at 10% of significance level through FDMA, CDMA, DFA, and R/S_{al} . While for the RUB/USD the null hypothesis of long memory can be rejected at 1% of significance by the CDMA and R/S_{al} approaches, which offers evidence supporting memory behaviors. Nevertheless, these results brightly contradict with the results of FDMA, BDMA, and DFA at 1% of significance which indicate the presence of

no long-term correlations when the whole series is considered. Besides, the results for INR/USD indicate that the null hypothesis can be rejected at the significance level of 5% through all the methods, which offers evidence that supports the persistence in the return series in accordance with Datta & Bhattacharyya(2018). Furthermore, we find that the null hypothesis of the efficiency in the Chinese exchange rate can be rejected at 5% of significant. Finally, at the significance level of 1% through all the different approaches, the null hypothesis cannot be rejected for the whole series in ZAR/USD which means that this series has been in weak form efficient during the whole period. Thus, in the South African forex market, investors could not have abnormal gains on returns through arbitraging and speculative activities. But also this implies that the current values of exchange rates ZAR/USD is respectively independent (Salisu & Ayinde, 2016).

To test whether the Global financial crisis induces changes in the Hurst exponents of the different series. First, we compare the results of Sub 1 for the pre-crisis and Sub 2 for the crisis periods. If the null hypothesis could not be rejected in Sub1 but it could reject for Sub 2, then we conclude that the financial crisis had a significant influence on the Hurst exponent. In this specific case, the Hurst exponent should not be considered different from 0,5 in Sub1 but it should be considered different from 0,5 in Sub2. (Horta et al., 2014). Through the analysis of the different approaches, we get mixed results. For this reason, we follow the recommendation by Weron (2002) and Xu et al. (2005). Weron (2002) compares between the R/S_{al} and DFA. The authors conclude that DFA outperforms R/S_{al} due to R/S estimates are influenced by the choice of minimum and maximum scale. The overestimation decreases significantly with growing length (Kristoufek, 2009). While Xu et al. (2005) compare the different versions of DMA versus the DFA. Their results suggest that CDMA produces better estimations when the Hurst exponent (H) is in the range of $0,3 < H < 0,7$ at small scales $n < 10^2$ and in $H > 0,7$ at intermediate scales $10^2 < n < 10^4$. However, the CDMA method strongly underestimates $H > 0,7$ at small scales $n < 10^2$, while the DFA method provides quite accurately the correlations of at both small and intermediate scales. Similar results are provided by Bashan et al. (2008) and Shao et al. (2012), the former mentions that CDMA is slightly superior to DFA in short data with weak trends while the later remarks that CDMA and DFA remain the best methods to get the Hurst index of time series.

Based on the analysis above, the time-varying efficiency for BRICS's exchange rates are going to be expressed in terms of DFA. In the pre-crisis period, the results are the followings: the RUB/USD, the CNY/USD and the ZAR/USD are the exchange rates that do not exhibit long memory at 1% of significance. Thus, the null hypothesis could not be rejected, confirming that these series are efficient. While in the

cases of the BRL/USD and the INR/USD, the null hypothesis is rejected at 10% and 1% level of significance, respectively. Similarly, Kumar & Pathak (2016) find inefficiency in the Indian forex market before the subprime crisis. Furthermore, in the period of the crisis (Sub2), the null hypothesis is rejected for RUB/USD, INR/USD and CNY/USD. The degree of efficiency is higher during this period suggesting vulnerability of these forex market to the mentioned shock. The results founded for INR/USD are in line with Khuntia et al. (2018). We additionally considered the Russian financial crisis (Sub4) in the series of RUB/USD (Table 6.2B). However, the results do not show evidence that support the AMH due to at the significance level of 0,01, the null hypothesis cannot be rejected. One can conclude that the EMH holds during this period. After the crisis, at 1% level of significance in the series of BRL/USD, RUB/USD and INR/USD the null hypothesis cannot be rejected. This confirms that post crisis period the returns of the exchange rate showed no signs of the long-range memory. While for the case of CNY/USD is found that the efficiency is not improved significantly, one of the factors related to this inefficiency is the reform of the Chinese RMB exchange rate flexibility on 22 June 2010 (Cao et al., 2018).

In general, these findings suggest that efficiency in the RUB/USD, CNY/USD and INR/USD market is a time-varying phenomenon influenced by the Global financial crisis which adheres to the fundamental notion of the AMH (Lo, 2004, 2005) while ZAR/USD is efficient and resilient to crisis (Salisu et al., 2016; Yaya et al., 2019).

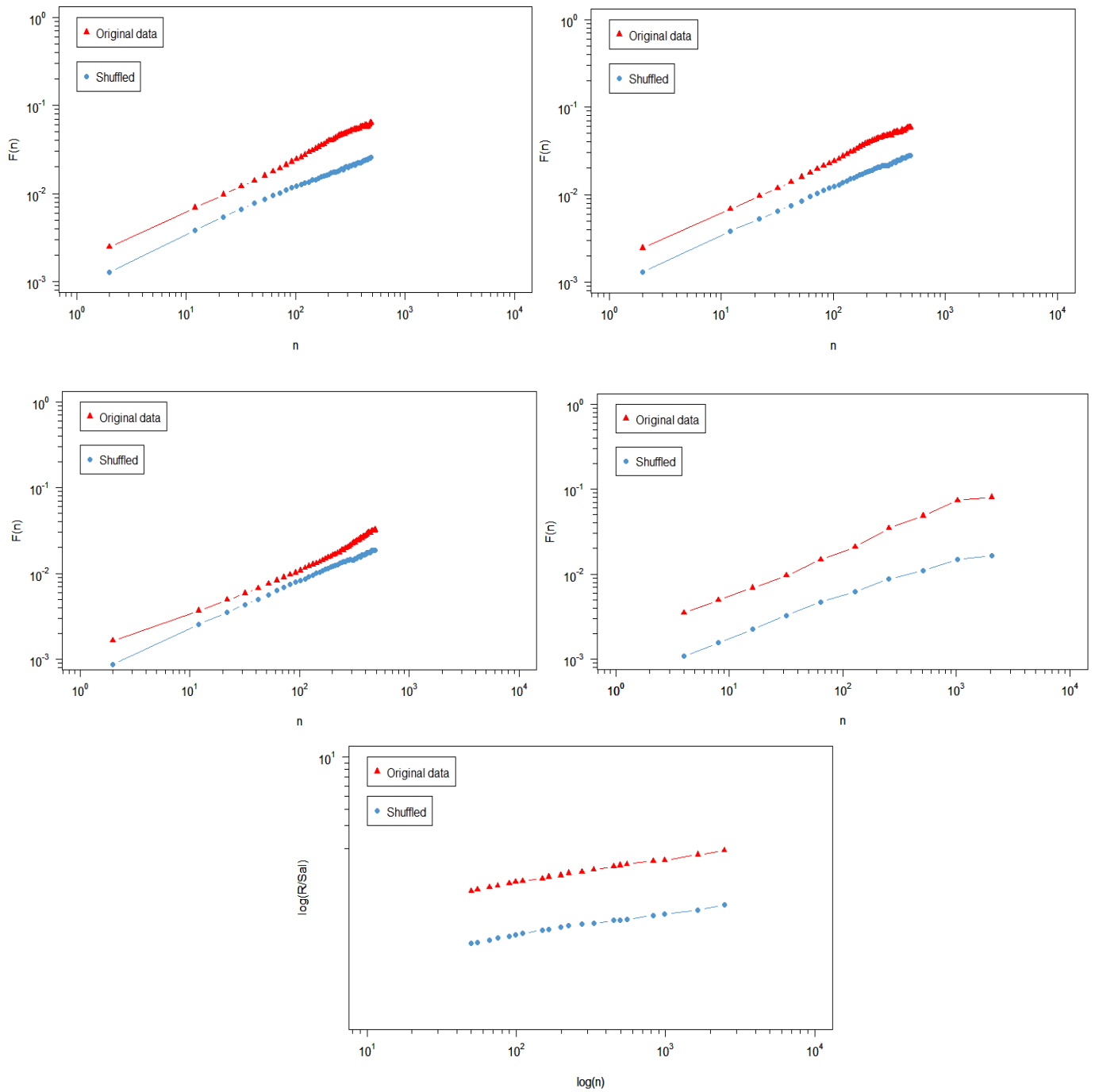


Figure 6.1 Plots of the fluctuation function $F(n)$ with respect to the box size n for the original series and shuffled series for the whole series. (A) FDMA, (B) BDMA, (C) CDMA, (D) DFA and (E) R/S_{all} for BRL/USD exchange rate

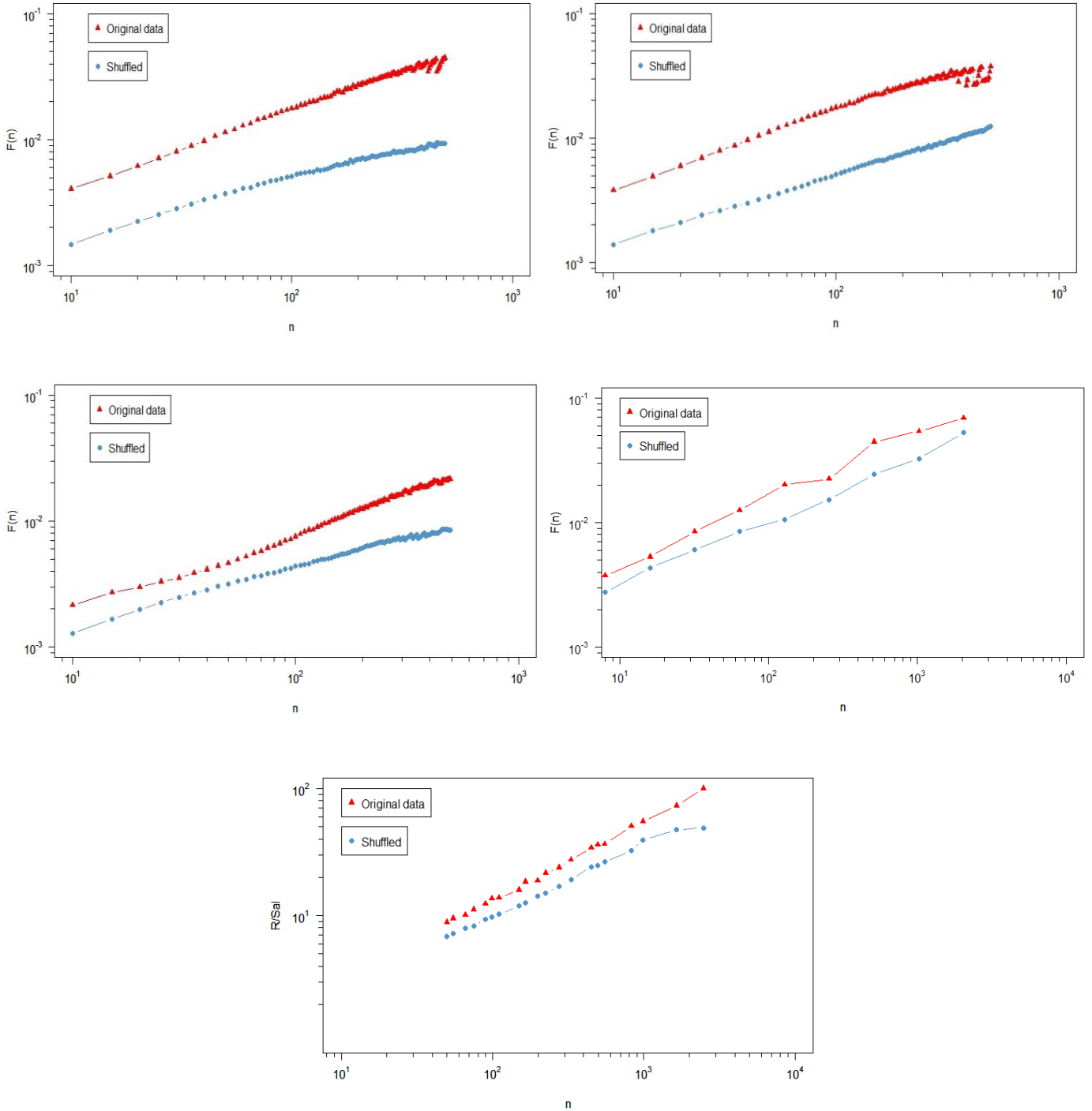


Figure 6.2 Plots of the fluctuation function $F(n)$ with respect to the box size n for the original series and shuffled series for the whole series. (A) FDMA, (B) BDMA, (C) CDMA, (D) DFA and (E) R/S_{al} for RUB/USD exchange rate.

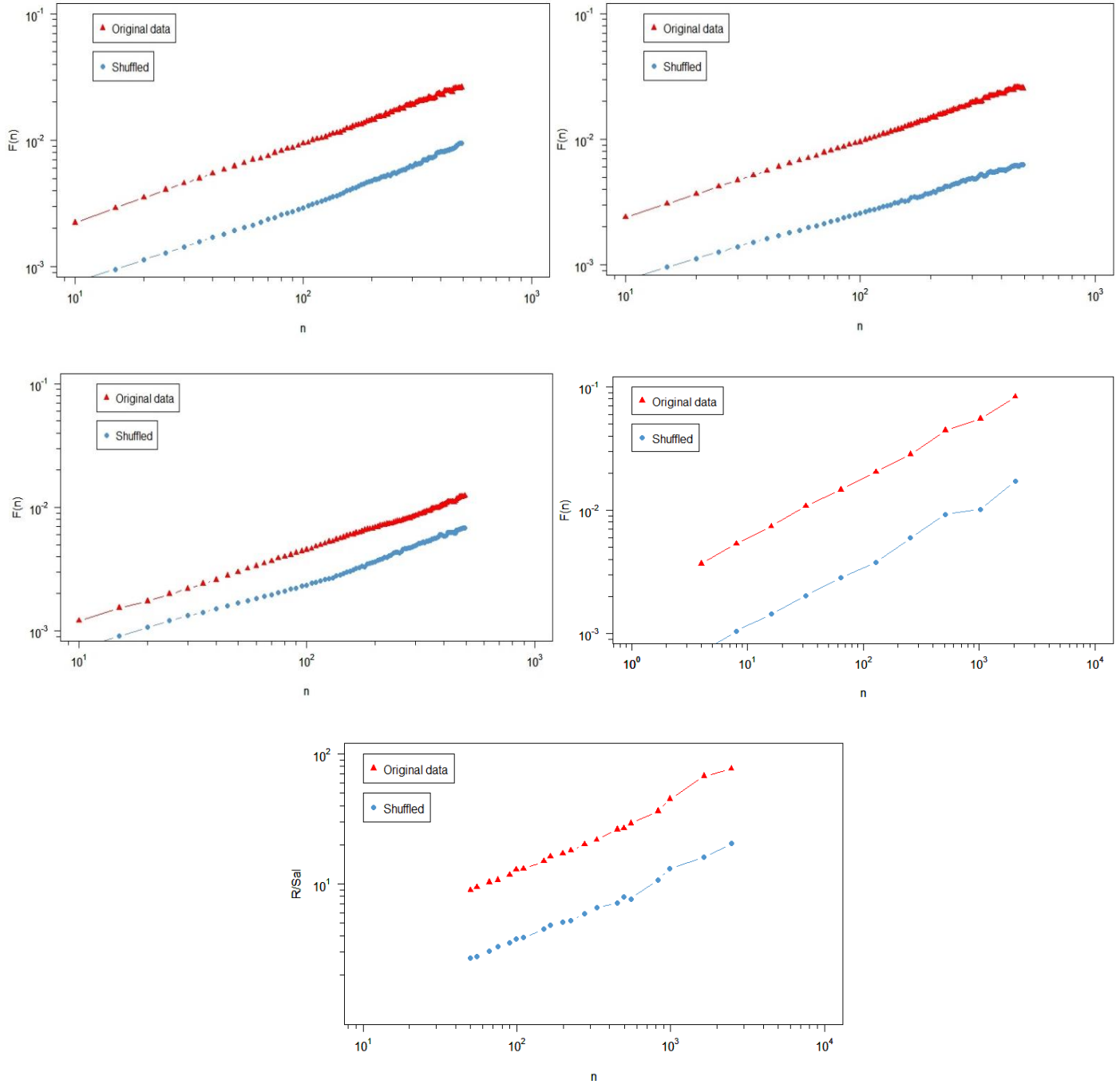


Figure 6.3 Plots of the fluctuation function $F(n)$ with respect to the box size n for the original series and shuffled series for the whole series. (A) FDMA, (B) BDMA, (C) CDMA (D) DFA and (E) R/S_{al} for INR/USD exchange rate.

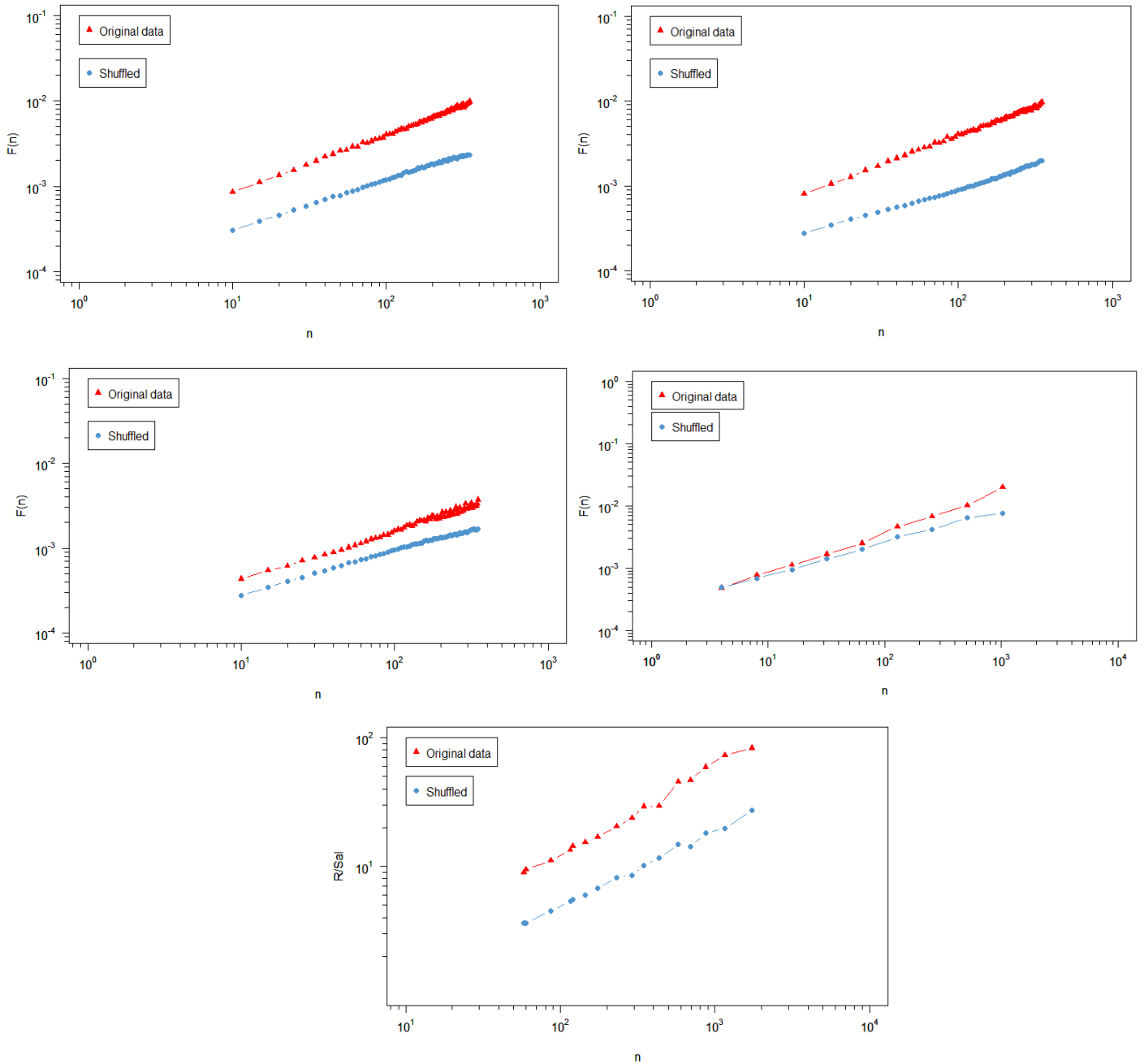


Figure 6.4 Plots of the fluctuation function $F(n)$ with respect to the box size n for the original series and shuffled series for the whole series. (A) FDMA, (B) BDMA, (C) CDMA (D) DFA and (E) R/S_{al} for CNY/USD exchange rate.

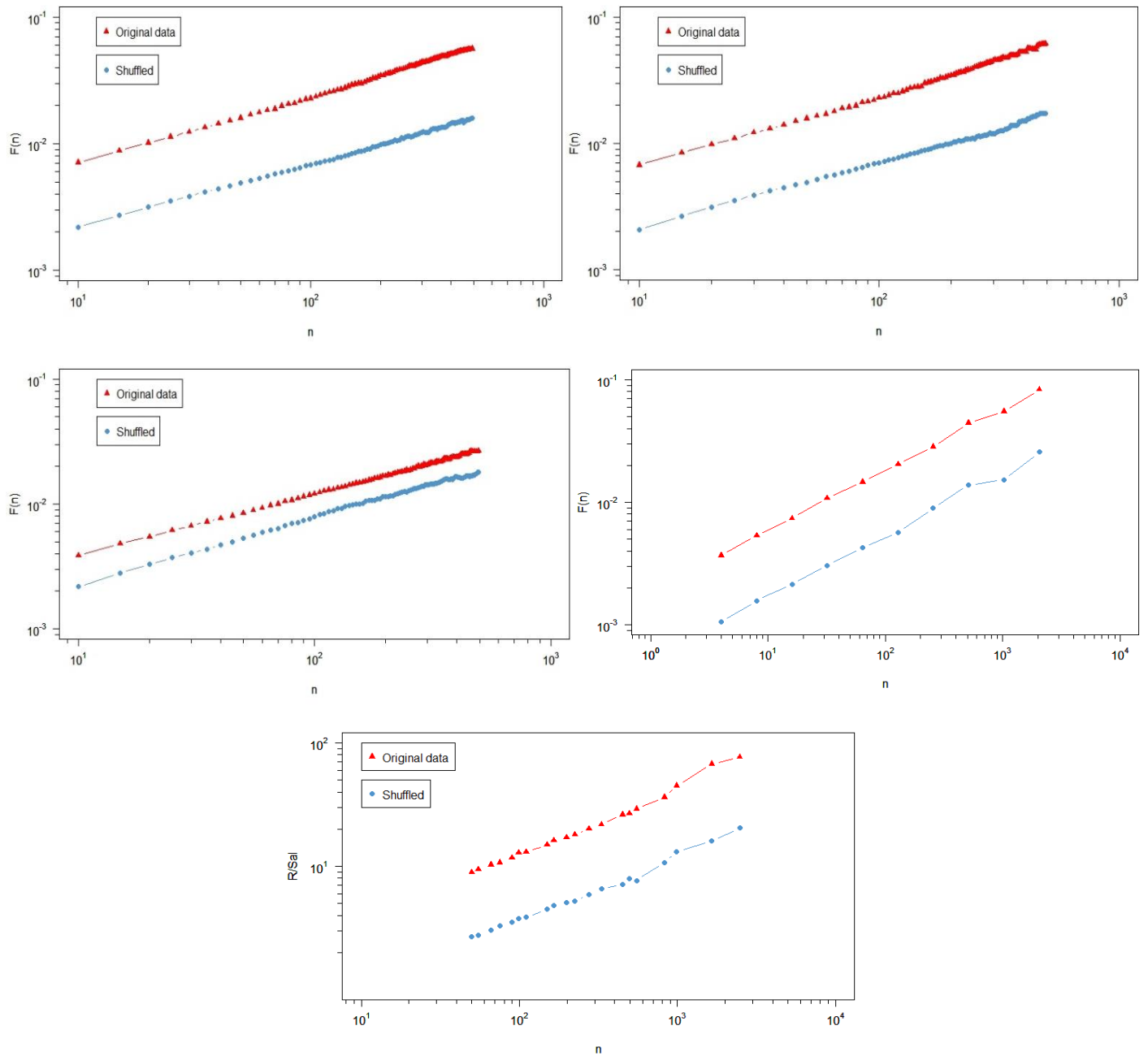


Figure 6.5 Plots of the fluctuation function $F(n)$ with respect to the box size n for the original series and shuffled series for the whole series. (A) FDMA, (B) BDMA, (C) CDMA, (D) DFA and (E) R/S_{al} for ZAR/USD exchange rate.

		Whole		Sub1		Sub2		Sub3
FDMA	H	0,613		0,678		0,647		0,590
	[Hs]	0,507		0,527		0,541		0,514
	p	0,044	**	0,030	**	0,194		0,200
BDMA	H	0,598		0,679		0,592		0,581
	[Hs]	0,515		0,532		0,546		0,522
	p	0,118		0,012	**	0,824		0,344
CDMA	H	0,613		0,543		0,466		0,526
	[Hs]	0,505		0,511		0,507		0,501
	p	0,002	***	0,436		0,452		0,522
DFA	H	0,549		0,564		0,560		0,524
	[Hs]	0,499		0,502		0,494		0,493
	p	0,096	*	0,056	*	0,212		0,360
R/Sal	H	0,559		0,492		0,589		0,543
	[Hs]	0,501		0,499		0,497		0,500
	p	0,056	*	0,826		0,306		0,326

* Denote the significance at 10% levels, ** at 5% levels, *** at 1% levels.

Table 6.1 Estimated Hurst exponents and bootstrapping test for BRL/USD exchange rate

		Whole		Sub1		Sub2		Sub3
FDMA	H	0,597		0,741		0,768		0,608
	[Hs]	0,523		0,530		0,560		0,538
	p	0,206		0,000	***	0,010	**	0,248
BDMA	H	0,522		0,758		0,746		0,578
	[Hs]	0,527		0,537		0,519		0,540
	p	0,918		0,000	***	0,340		0,552
CDMA	H	0,650		0,592		0,649		0,592
	[Hs]	0,523		0,514		0,520		0,522
	p	0,000	***	0,048	**	0,676		0,000 ***
DFA	H	0,540		0,550		0,594		0,531
	[Hs]	0,498		0,502		0,498		0,499
	p	0,082	*	0,154		0,084	*	0,336
R/Sal	H	0,620		0,621		0,673		0,538
	[Hs]	0,505		0,502		0,493		0,498
	p	0,000	***	0,000	***	0,018	**	0,358

* Denote the significance at 10% levels, ** at 5% levels, *** at 1% levels.

Table 6.2A Estimated Hurst exponents and bootstrapping test for RUB /USD exchange rate

		Whole	Sub1	Sub2	Sub3	Sub4	Sub5
FDMA	H	0,597	0,741	0,768	0,593	0,790	0,521
	[Hs]	0,523	0,530	0,560	0,535	0,544	0,535
	p	0,206	0,000 ***	0,010 **	0,406	0,038 **	0,872
BDMA	H	0,522	0,758	0,746	0,591	0,660	0,566
	[Hs]	0,527	0,537	0,519	0,590	0,550	0,571
	p	0,918	0,000 ***	0,340	0,514	0,362	0,940
CDMA	H	0,650	0,592	0,649	0,533	0,451	0,424
	[Hs]	0,523	0,514	0,520	0,511	0,505	0,514
	p	0,000 ***	0,048 **	0,676	0,624	0,492	0,094 *
DFA	H	0,540	0,550	0,594	0,522	0,544	0,503
	[Hs]	0,498	0,502	0,498	0,499	0,490	0,498
	p	0,082 *	0,154	0,084 *	0,574	0,456	0,906
R/Sal	H	0,620	0,621	0,673	0,552	0,629	0,516
	[Hs]	0,505	0,502	0,493	0,530	0,497	0,499
	p	0,000 ***	0,000 ***	0,018 **	0,505	0,106	0,822

* Denote the significance at 10% levels, ** at 5% levels, *** at 1% levels.

Table 6.2B Estimated Hurst exponents and bootstrapping test for RUB /USD exchange rate

		Whole	Sub1	Sub2	Sub3
FDMA	H	0,621	0,669	0,624	0,542
	[Hs]	0,503	0,526	0,533	0,513
	p	0,028 **	0,026 **	0,027 **	0,648
BDMA	H	0,635	0,699	0,594	0,538
	[Hs]	0,517	0,532	0,526	0,523
	p	0,030 **	0,002 ***	0,820	0,814
CDMA	H	0,597	0,620	0,613	0,585
	[Hs]	0,505	0,513	0,503	0,503
	p	0,002 ***	0,006 ***	0,064 *	0,040 **
DFA	H	0,537	0,550	0,638	0,525
	[Hs]	0,498	0,501	0,506	0,505
	p	0,022 **	0,000 ***	0,000 ***	0,400
R/Sal	H	0,588	0,683	0,537	0,579
	[Hs]	0,503	0,501	0,504	0,499
	p	0,004 ***	0,000 ***	0,698	0,074 *

* Denote the significance at 10% levels, ** at 5% levels, *** at 1% levels.

Table 6.3 Estimated Hurst exponents and bootstrapping test for INR/USD exchange rate

		Whole	Sub1	Sub2	Sub3
FDMA	H	0,672	0,539	0,816	0,618
	[Hs]	0,511	0,534	0,540	0,526
	p	0,002 ***	0,958	0,000 ***	0,168
BDMA	H	0,680	0,587	0,790	0,600
	[Hs]	0,521	0,548	0,527	0,529
	p	0,004 **	0,686	0,266	0,240
CDMA	H	0,589	0,290	0,481	0,623
	[Hs]	0,513	0,496	0,509	0,511
	p	0,030 **	0,000 ***	0,650	0,000 ***
DFA	H	0,641	0,514	0,670	0,590
	[Hs]	0,499	0,494	0,496	0,497
	p	0,000 ***	0,726	0,000 ***	0,012 **
R/Sal	H	0,687	0,370	0,591	0,606
	[Hs]	0,508	0,497	0,496	0,528
	p	0,000 ***	0,164	0,316	0,058 *

* Denote the significance at 10% levels, ** at 5% levels, *** at 1% levels.

Table 6.4 Estimated Hurst exponents and bootstrapping test for CNY/USD exchange rate

		Whole	Sub1	Sub2	Sub3
FDMA	H	0,549	0,602	0,618	0,449
	[Hs]	0,503	0,519	0,540	0,517
	p	0,394	0,174	0,336	0,274
BDMA	H	0,581	0,623	0,590	0,424
	[Hs]	0,512	0,529	0,527	0,521
	p	0,228	0,176	0,816	0,126
CDMA	H	0,496	0,550	0,479	0,428
	[Hs]	0,502	0,504	0,498	0,500
	p	0,860	0,276	0,748	0,058 *
DFA	H	0,518	0,547	0,552	0,449
	[Hs]	0,500	0,499	0,496	0,497
	p	0,790	0,170	0,314	0,170
R/Sal	H	0,548	0,571	0,468	0,453
	[Hs]	0,498	0,496	0,487	0,528
	p	0,106	0,120	0,852	0,061 *

* Denote the significance at 10% levels, ** at 5% levels, *** at 1% levels.

Table 6.5 Estimated Hurst exponents and bootstrapping test for ZAR/USD exchange rate

7. Conclusion

In this study we have examined the impact of the Global financial crisis based on the estimation of Hurst exponent by means of R/S_{al} , DFA and DMA approaches to represent the dollar-based exchange rates of the BRICS countries together with a strict statistical test to verify the determinacy of each Hurst exponent. The aim was to understand the behavior of the Hurst exponent of the five-forex market over time and evaluate their efficiency. Over the full sample period using daily data from 2000 to 2018, we find evidence of weak persistence in almost all the returns, except for ZAR/USD. Furthermore, we have divided the whole period in three subperiods to identify changes in efficiency over time. For the analysis of time-varying efficiency the DFA approach was chosen due to it is the most suitable for small and intermediate scales. In this sense, the results can be summarized in three points. Firstly, most of the market returns (RUB/USD, CNY/USD and ZAR/USD) do not exhibit long memory in the pre-crisis period, suggesting that these forex markets tend to be efficient in tranquil periods. Secondly, the returns of RUB/USD, INR/USD and CNY/USD exhibit long memory during the Subprime crisis, suggesting that markets lost efficiency during this period. Thirdly, we evidence that RUB/USD and INR/USD moved from long memory and persistence towards efficiency, while for the returns of CNY/USD the efficiency has not improved significantly. Thus, we find evidence of time-varying degree of efficiency which supports the AMH in the RUB/USD, CNY/USD and INR/USD.

8. Bibliography

- Abounoori, E., Shahrazi, M., & Rasekhi, S. (2012). An investigation of Forex market efficiency based on detrended fluctuation analysis: A case study for Iran. *Physica A: Statistical Mechanics and Its Applications*, 391(11), 3170–3179. <https://doi.org/10.1016/j.physa.2011.12.045>
- Al-Khazali, O. M., Pyun, C. S., & Kim, D. (2012). Are exchange rate movements predictable in Asia-Pacific markets? Evidence of random walk and martingale difference processes. *International Review of Economics & Finance*, 21(1), 221–231. <https://doi.org/10.1016/j.iref.2011.07.002>
- Almail, A., & Almudhaf, F. (2017). Adaptive Market Hypothesis: Evidence from three centuries of UK data. *Oviedo University Press Economics and Business Letters*, 6(2), 48–53. <https://doi.org/10.17811/ebl.6.2.2017.48-53>
- Aloui, R., Aïssa, M. S. Ben, & Nguyen, D. K. (2011). Global financial crisis, extreme interdependences, and contagion effects: The role of economic structure? *Journal of Banking & Finance*, 35(1), 130–141. <https://doi.org/10.1016/j.jbankfin.2010.07.021>
- Ang, A., & Timmermann, A. (2012). Regime Changes and Financial Markets. *Annual Review of Financial Economics*, 4(1), 313–337. <https://doi.org/10.1146/annurev-financial-110311-101808>
- Anis, A. A., & Lloyd, E. H. (1976). The Expected Value of the Adjusted Rescaled Hurst Range of Independent Normal Summands. *Biometrika*, 63(1), 111. <https://doi.org/10.2307/2335090>
- Arianos, S., & Carbone, A. (2007). Detrending moving average algorithm: A closed-form approximation of the scaling law. *Physica A: Statistical Mechanics and Its Applications*, 382(1), 9–15. <https://doi.org/10.1016/j.physa.2007.02.074>
- Aroskar, R., Sarkar, S. K., & Swanson, P. E. (2004). European foreign exchange market efficiency: Evidence based on crisis and noncrisis periods. *International Review of Financial Analysis*, 13(3), 333–347. <https://doi.org/10.1016/j.irfa.2004.02.011>
- Azad, A. S. M. S. (2009). Random walk and efficiency tests in the Asia-Pacific foreign exchange markets: Evidence from the post-Asian currency crisis data. *Research in International Business and Finance*, 23(3), 322–338. <https://doi.org/10.1016/j.ribaf.2008.11.001>
- Baker, H. K., Filbeck, G., & Ricciardi, V. (2017). *Financial Behavior: Players, Services, Products, and Markets*. Oxford University Press.
- Bashan, A., Bartsch, R., Kantelhardt, J. W., & Havlin, S. (2008). Comparison of detrending methods for fluctuation analysis. *Physica A: Statistical Mechanics and Its Applications*, 387(21), 5080–5090. <https://doi.org/10.1016/j.physa.2008.04.023>
- Belaire-Franch, J., & Contreras, D. (2004). *Ranks and signs-based multiple variance ratio tests*.
- Belaire-Franch, J., & Opong, K. K. (2005). Some evidence of random walk behavior of Euro exchange rates using ranks and signs. *Journal of Banking & Finance*, 29(7), 1631–1643. <https://doi.org/10.1016/j.jbankfin.2004.06.031>
- Boya, C. M. (2019). From efficient markets to adaptive markets: Evidence from the French stock exchange. *Research in International Business and Finance*, 49(October 2018), 156–165. <https://doi.org/10.1016/j.ribaf.2019.03.005>

- Cajueiro, D. O., Gogas, P., & Tabak, B. M. (2009). Does financial market liberalization increase the degree of market efficiency? The case of the Athens stock exchange. *International Review of Financial Analysis*, 18(1–2), 50–57. <https://doi.org/10.1016/j.irfa.2008.11.004>
- Campbell, J., Lo, A., & Mackinlay, A. (1997). *The Econometrics of Financial Markets*. (N. . Princeton, Ed.). Princeton University Press.
- Cao, G., He, L.-Y., & Cao, J. (2018). Long Memory Methods and Comparative Analysis. In *Multifractal Detrended Analysis Method and Its Application in Financial Markets* (pp. 7–20). Singapore: Springer Singapore. https://doi.org/10.1007/978-981-10-7916-0_2
- Charles, A., Darné, O., & Kim, J. H. (2012). Exchange-rate return predictability and the adaptive markets hypothesis: Evidence from major foreign exchange rates. *Journal of International Money and Finance*, 31(6), 1607–1626. <https://doi.org/10.1016/j.jimonfin.2012.03.003>
- Chiang, S.-M., Lee, Y.-H., Su, H.-M., & Tzou, Y.-P. (2010). Efficiency tests of foreign exchange markets for four Asian Countries. *Research in International Business and Finance*, 24(3), 284–294. <https://doi.org/10.1016/j.ribaf.2010.01.001>
- Couillard, M., & Davison, M. (2005). A comment on measuring the Hurst exponent of financial time series. *Physica A: Statistical Mechanics and Its Applications*, 348, 404–418. <https://doi.org/10.1016/j.physa.2004.09.035>
- Da Silva, S., Matsushita, R., Gleria, I., & Figueiredo, A. (2007). Hurst exponents, power laws, and efficiency in the Brazilian foreign exchange market. *Economics Bulletin*, 7(1).
- Datta, R. P., & Bhattacharyya, R. (2018). Has the efficiency of foreign exchange markets in India evolved over time? *International Journal of Emerging Markets*, 13(4), 676–688. <https://doi.org/10.1108/IJoEM-05-2017-0161>
- Domínguez, M. A., & Lobato, I. N. (2003). Testing the Martingale Difference Hypothesis. *Econometric Reviews*, 22(4), 351–377. <https://doi.org/10.1081/ETC-120025895>
- Escanciano, J. C., & Velasco, C. (2006). Generalized spectral tests for the martingale difference hypothesis. *Journal of Econometrics*, 134(1), 151–185. <https://doi.org/10.1016/j.jeconom.2005.06.019>
- Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance*, 25(2), 383. <https://doi.org/10.2307/2325486>
- Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics*, 14(3), 319–338. [https://doi.org/10.1016/0304-3932\(84\)90046-1](https://doi.org/10.1016/0304-3932(84)90046-1)
- Frankel, J., & Sarvelos, G. (2012). Can leading indicators assess country vulnerability? Evidence from the 2008–09 global financial crisis. *Journal of International Economics*, 87(2), 216–231. <https://doi.org/10.1016/j.jinteco.2011.12.009>
- Frömmel, M. (2017). *Finance 2: Asset Allocation and Market Efficiency* (2nd ed.). Books on Demand.
- Ghazani, M., & Araghi, M. (2014). Evaluation of the adaptive market hypothesis as an evolutionary perspective on market efficiency: Evidence from the Tehran stock exchange. *Research in International Business and Finance*, 32, 50–59. <https://doi.org/10.1016/j.ribaf.2014.03.002>

- Guangxi, C., Yan, H., & Weijun, C. (2014). Time-Varying Long Memories of the Chinese Currency and Stock Markets Based on the Hurst Exponent. *Fluctuation and Noise Letters*, 13(01), 1450007. <https://doi.org/10.1142/s0219477514500072>
- Gupta, R., & Plakandaras, V. (2018). *Efficiency in BRICS Currency Markets using Long-Spans of Data: Evidence from Model-Free Tests of Directional Predictability*.
- Hallwood, P., & MacDonald, R. (1994). *International money and finance*. (M. Cambridge, Ed.) (2nd ed.). Oxford, UK.
- Hiremath, G. S., & Narayan, S. (2016). Testing the adaptive market hypothesis and its determinants for the Indian stock markets. *Finance Research Letters*, 19, 173–180. <https://doi.org/10.1016/j.frl.2016.07.009>
- Horta, P. (2013). Contagion effects in the European NYSE Euronext stock markets in the context of the 2010 sovereign debt crisis. *Investment Management and Financial Innovations*, 10(4), 114–124.
- Horta, P., Lagoa, S., & Martins, L. (2014). The impact of the 2008 and 2010 financial crises on the Hurst exponents of international stock markets: Implications for efficiency and contagion. *International Review of Financial Analysis*, 35, 140–153. <https://doi.org/10.1016/j.irfa.2014.08.002>
- Hull, M., & McGroarty, F. (2014). Do emerging markets become more efficient as they develop? Long memory persistence in equity indices. *Emerging Markets Review*, 18, 45–61. <https://doi.org/10.1016/j.ememar.2013.11.001>
- Hurst, H. E. (1951). *Transactions of the American Society of Civil Engineers*, (116), 770–799.
- Ilzetzki, E., Reinhart, C., & Rogoff, K. S. (2019). Exchange Arrangements Entering the 21st Century: Which Anchor Will Hold? *Quarterly Journal of Economics*, 2(134), 599–646. <https://doi.org/10.1093/qje/qjy033>. Advance
- International Monetary Fund. (2019). *Exchange Arrangements and Exchange Restrictions*. *Exchange Organizational Behavior Teaching Journal*.
- Ito, M., & Sugiyama, S. (2009). Measuring the degree of time varying market inefficiency. *Economics Letters*, 103(1), 62–64. <https://doi.org/10.1016/j.econlet.2009.01.028>
- Jagric, T., Podobnik, B., & Kolanovic, M. (2005). Does the Efficient Market Hypothesis Hold?: Evidence from Six Transition Economies. *Eastern European Economics*, 43(4), 79–103. <https://doi.org/10.1080/00128775.2005.11041112>
- Jeon, B. N., & Seo, B. (2003). The impact of the Asian financial crisis on foreign exchange market efficiency: The case of East Asian countries. *Pacific-Basin Finance Journal*, 11(4), 509–525. [https://doi.org/10.1016/S0927-538X\(03\)00052-0](https://doi.org/10.1016/S0927-538X(03)00052-0)
- Jiang, Z. Q., Xie, W. J., & Zhou, W. X. (2014). Testing the weak-form efficiency of the WTI crude oil futures market. *Physica A: Statistical Mechanics and Its Applications*, 405, 235–244. <https://doi.org/10.1016/j.physa.2014.02.042>
- Karmakar, A. K., & Sovik, M. (2017). Global Financial Crisis and Its Ramifications on Capital Markets. In Ü. Hacıoğlu & H. Dincer (Eds.) (pp. 175–190). Cham: Springer International Publishing. <https://doi.org/10.1007/978-3-319-47021-4>

- Katusiime, L., Shamsuddin, A., & Agbola, F. W. (2015). Foreign exchange market efficiency and profitability of trading rules: Evidence from a developing country. *International Review of Economics and Finance*, 35, 315–332. <https://doi.org/10.1016/j.iref.2014.10.003>
- Khuntia, S., Pattanayak, J. K., & Hiremath, G. S. (2018). Is The Foreign Exchange Market Efficiency Adaptive? The Empirical Evidence From India. *Journal of Asia-Pacific Business*, 19(04), 1–25. <https://doi.org/10.1080/10599231.2018.1525249>
- Kim, J. H. (2009). Automatic variance ratio test under conditional heteroskedasticity. *Finance Research Letters*, 6(3), 179–185. <https://doi.org/10.1016/j.frl.2009.04.003>
- Kim, J. H., Shamsuddin, A., & Lim, K.-P. (2011). Stock return predictability and the adaptive markets hypothesis: Evidence from century-long U.S. data. *Journal of Empirical Finance*, 18(5), 868–879. <https://doi.org/10.1016/j.jempfin.2011.08.002>
- Kristoufek, L. (2009). R/S analysis and DFA: finite sample properties and confidence intervals, (16446), 11. Retrieved from <http://mpa.ub.uni-muenchen.de/16446/>
- Kumar, D. (2018). Market Efficiency in Indian Exchange Rates: Adaptive Market Hypothesis. *Theoretical Economics Letters*, 08(09), 1582–1598. <https://doi.org/10.4236/tel.2018.89101>
- Kumar, S., & Pathak, R. (2016). Do the calendar anomalies still exist? Evidence from Indian currency market. *Managerial Finance*, 42(2), 136–150. <https://doi.org/10.1108/MF-05-2015-0146>
- Lahmiri, S. (2015). Long memory in international financial markets trends and short movements during 2008 financial crisis based on variational mode decomposition and detrended fluctuation analysis. *Physica A: Statistical Mechanics and Its Applications*, 437, 130–138. <https://doi.org/10.1016/j.physa.2015.05.067>
- Lazăr, D., Todea, A., & Filip, D. (2012). Martingale difference hypothesis and financial crisis: Empirical evidence from European emerging foreign exchange markets. *Economic Systems*, 36(3), 338–350. <https://doi.org/10.1016/j.ecosys.2012.02.002>
- Levich, R. M. (1985). Chapter 19 Empirical studies of exchange rates: Price behavior, rate determination and market efficiency. In *Handbook of International Economics* (Vol. 2, pp. 979–1040). Elsevier Science Publishers B.V. [https://doi.org/10.1016/S1573-4404\(85\)02010-X](https://doi.org/10.1016/S1573-4404(85)02010-X)
- Levich, R. M. (2001). The Importance of Emerging Capital Markets, 45, 1–45.
- Linton, O., & Whang, Y.-J. (2007). The quantilogram: With an application to evaluating directional predictability. *Journal of Econometrics*, 141(1), 250–282. <https://doi.org/10.1016/j.jeconom.2007.01.004>
- Lo, A. W. (2004). The Adaptive Markets Hypothesis. *The Journal of Portfolio Management*, 30(5), 15–29. <https://doi.org/10.3905/jpm.2004.442611>
- Lo, A. W. (2005). Reconciling efficient markets with behavioral finance: The adaptive markets hypothesis. *Journal of Investment Consulting*, 7(2), 21–44. Retrieved from <https://www.semanticscholar.org/paper/Reconciling-Efficient-Markets-with-Behavioral-Lo/5f00896bbf9afb7725f9ae3bd62b48d3c317c247>
- Lo, A. W. (2012). Adaptive Markets and the New World Order (corrected May 2012) . *Financial Analysts Journal*, 68(2), 18–29. <https://doi.org/10.2469/faj.v68.n2.6>

- Lo, A. W. (2017). *Adaptive Markets: Financial Evolution at the Speed of Thought*. (Princeton University Press, Ed.). Princeton University Press.
- Ma, P., Li, D., & Li, S. (2016). Efficiency and cross-correlation in equity market during global financial crisis: Evidence from China. *Physica A: Statistical Mechanics and Its Applications*, 444, 163–176. <https://doi.org/10.1016/j.physa.2015.10.019>
- Mandelbrot, B. (1972). Statistical Methodology for Nonperiodic Cycles: From the Covariance to R/S Analysis. In The National Bureau of Economic Research (Ed.), *NBER* (Vol. 1, pp. 259–290). The National Bureau of Economic Research. Retrieved from <http://www.nber.org/books/aesm72-3>
- Mandelbrot, B. B., & Wallis, J. R. (1969). Some long-run properties of geophysical records. *Water Resources Research*, 5(2), 321–340. <https://doi.org/10.1029/WR005i002p00321>
- Menkhoff, L. (1998). The noise trading approach — questionnaire evidence from foreign exchange. *Journal of International Money and Finance*, 17(3), 547–564. [https://doi.org/10.1016/S0261-5606\(98\)00016-3](https://doi.org/10.1016/S0261-5606(98)00016-3)
- Mynhardt, R. H., Plastun, A., & Makarenko, I. (2014). Behavior of financial markets efficiency during the financial market crisis: 2007 – 2009. *Corporate Ownership and Control*, 11(2), 473–485. <https://doi.org/10.22495/cocv11i2c5p4>
- Neely, C. J., Weller, P. A., & Ulrich, J. M. (2009). The adaptive markets hypothesis: Evidence from the foreign exchange market. *Journal of Financial and Quantitative Analysis*, 44(2), 467–488. <https://doi.org/10.1017/S0022109009090103>
- Ning, Y., Wang, Y., & Su, C. (2017). How Did China's Foreign Exchange Reform Affect the Efficiency of Foreign Exchange Market ? *Physica A*. <https://doi.org/10.1016/j.physa.2017.04.150>
- Olufemi, A. P., Adewale, A. O., & Oseko, M. S. (2017). Efficiency of Foreign Exchange Markets in Sub-Saharan Africa in the Presence of Structural Break: A Linear and Non-Linear Testing Approach. *Journal of Economics and Behavioral Studies*, 9(4), 122. <https://doi.org/10.22610/jebbs.v9i4.1827>
- Onali, E., & Goddard, J. (2011). Are European equity markets efficient? New evidence from fractal analysis. *International Review of Financial Analysis*, 20(2), 59–67. <https://doi.org/10.1016/j.irfa.2011.02.004>
- Peng, C.-K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., & Goldberger, A. L. (1994). Mosaic organization of DNA nucleotides. *Physical Review E*, 49(2), 1685–1689. <https://doi.org/10.1103/PhysRevE.49.1685>
- Peng, C. -K., Havlin, S., Stanley, H. E., & Goldberger, A. L. (1995). Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 5(1), 82–87. <https://doi.org/10.1063/1.166141>
- Peters, E. E. (1994). *Applying Chaos Theory To Investment And Economics*. (John Wiley & Sons). Retrieved from https://books.google.com.my/books/about/Fractal_Market_Analysis.html?id=_bkoySKyc_cC&printsec=frontcover&source=kp_read_button&redir_esc=y#v=onepage&q&f=false
- Qin, J., Lu, X., Zhou, Y., & Qu, L. (2015). The effectiveness of China's RMB exchange rate reforms: An insight from multifractal detrended fluctuation analysis. *Physica A: Statistical Mechanics and Its Applications*, 421(1), 443–454. <https://doi.org/10.1016/j.physa.2014.11.053>

- Rashid, A. (2006). Do exchange rates follow random walks? An Application of Variance-Ratio Test. *Pakistan Economic and Social Review*, 44(1), 57–79. Retrieved from <http://www.jstor.org/stable/25825284>
- Rizvi, S. A. R., & Arshad, S. (2016). How does crisis affect efficiency? An empirical study of East Asian markets. *Borsa Istanbul Review*, 16(1), 1–8. <https://doi.org/10.1016/j.bir.2015.12.003>
- Rossi, B. (2013). Exchange Rate Predictability. *SSRN Electronic Journal*. <https://doi.org/10.2139/ssrn.2316312>
- Russian Institute for Strategic Studies. (2017). *Use of National Currencies in International Settlements. Experience of the Brics Countries*. Retrieved from http://www.nkibrics.ru/system/asset_publications/data/5a4f/6950/6272/695d/4717/0000/original/Use_of_national_currencies_in_international_settlements._Experience_of_the_BRICS_countries.pdf?1515153744
- Ruzima, M., & Boachie, M. K. (2018). Exchange rate uncertainty and private investment in BRICS economies. *Asia-Pacific Journal of Regional Science*, 2(1), 65–77. <https://doi.org/10.1007/s41685-017-0062-0>
- Salisu, A. A., & Ayinde, T. O. (2016). Testing the Martingale Difference Hypothesis (MDH) with Structural Breaks: Evidence from Foreign Exchanges of Nigeria and South Africa. *Journal of African Business*, 17(3), 342–359. <https://doi.org/10.1080/15228916.2016.1183274>
- Salisu, A. A., Oloko, T. F., & Oyewole, O. J. (2016). Testing for martingale difference hypothesis with structural breaks: Evidence from Asia–Pacific foreign exchange markets. *Borsa Istanbul Review*, 16(4), 210–218. <https://doi.org/10.1016/j.bir.2016.09.001>
- Sánchez, M. A., Trinidad, J. E., & García, J. (2008). Some comments on Hurst exponent and the long memory processes on capital markets. *Physica A: Statistical Mechanics and Its Applications*, 387(22), 5543–5551. <https://doi.org/10.1016/j.physa.2008.05.053>
- Sensoy, A., & Tabak, B. M. (2016). Dynamic efficiency of stock markets and exchange rates. *International Review of Financial Analysis*, 47, 353–371. <https://doi.org/10.1016/j.irfa.2016.06.001>
- Shao, Y.-H., Gu, G.-F., Jiang, Z.-Q., Zhou, W.-X., & Sornette, D. (2012). Comparing the performance of FA, DFA and DMA using different synthetic long-range correlated time series. *Scientific Reports*, 2(1), 835. <https://doi.org/10.1038/srep00835>
- Shi, H.-L., Jiang, Z.-Q., & Zhou, W.-X. (2016). Time-varying return predictability in the Chinese stock market, 1(1), 1–11. <https://doi.org/10.1142/S2424942417400023>
- Soufian, M., Forbes, W., & Hudson, R. (2014). Adapting financial rationality: Is a new paradigm emerging? *Critical Perspectives on Accounting*, 25(8), 724–742. <https://doi.org/10.1016/j.cpa.2013.08.009>
- Stošić, D., Stošić, D., Stošić, T., & Stanley, H. E. (2015). Multifractal analysis of managed and independent float exchange rates. *Physica A: Statistical Mechanics and Its Applications*, 428, 13–18. <https://doi.org/10.1016/j.physa.2015.02.055>
- Tabak, B. M., & Cajueiro, D. O. (2007). Are the crude oil markets becoming weakly efficient over time? A test for time-varying long-range dependence in prices and volatility. *Energy Economics*, 29(1), 28–36. <https://doi.org/10.1016/j.eneco.2006.06.007>

- Tsangarides, C. G. (2012). Crisis and recovery: Role of the exchange rate regime in emerging market economies. *Journal of Macroeconomics*, 34(2), 470–488.
<https://doi.org/10.1016/j.jmacro.2012.01.005>
- Urquhart, A., & McGroarty, F. (2014). Calendar effects, market conditions and the Adaptive Market Hypothesis: Evidence from long-run U.S. data. *International Review of Financial Analysis*, 35, 154–166. <https://doi.org/10.1016/j.irfa.2014.08.003>
- Vieito, J. P., Wong, W. K., & Zhu, Z. Z. (2016). Could the global financial crisis improve the performance of the G7 stocks markets? *Applied Economics*, 48(12), 1066–1080.
<https://doi.org/10.1080/00036846.2015.1093083>
- Wang, Y., Liu, L., & Gu, R. (2009). Analysis of efficiency for Shenzhen stock market based on multifractal detrended fluctuation analysis. *International Review of Financial Analysis*, 18(5), 271–276.
<https://doi.org/10.1016/j.irfa.2009.09.005>
- Weron, R. (2002). Estimating long-range dependence: finite sample properties and confidence intervals. *Physica A: Statistical Mechanics and Its Applications*, 312(1–2), 285–299.
[https://doi.org/10.1016/S0378-4371\(02\)00961-5](https://doi.org/10.1016/S0378-4371(02)00961-5)
- Wilson, D., & Purushothaman, R. (2003). Dreaming With BRICs: The Path to 2050. *Goldman Sachs Global Economics*, 99.
- Xu, L., Ivanov, P. C., Hu, K., Chen, Z., Carbone, A., & Stanley, H. E. (2005). Quantifying signals with power-law correlations: A comparative study of detrended fluctuation analysis and detrended moving average techniques. *Physical Review E*, 71(5), 051101.
<https://doi.org/10.1103/PhysRevE.71.051101>
- Yamani, E. A.-T. (2017). Foreign Exchange Market Efficiency: Different Tales from Developed and Developing Markets during the Recent Financial Crisis. *SSRN Electronic Journal*, 1–49.
<https://doi.org/10.2139/ssrn.2972163>
- Yang, J., Su, X., & Kolari, J. W. (2008). Do Euro exchange rates follow a martingale? Some out-of-sample evidence. *Journal of Banking & Finance*, 32(5), 729–740.
<https://doi.org/10.1016/j.jbankfin.2007.05.009>
- Yang, Y.-H., Shao, Y.-H., Shao, H.-L., & Stanley, H. E. (2019). Revisiting the weak-form efficiency of the EUR/CHF exchange rate market: Evidence from episodes of different Swiss franc regimes. *Physica A: Statistical Mechanics and Its Applications*, 523, 734–746.
<https://doi.org/10.1016/j.physa.2019.02.056>
- Yaya, O. S., Saka, L., & Akanbi, O. B. (2019). Assessing Market Efficiency And Volatility Of Exchange Rates in South Africa and United Kingdom: Analysis Using Hurst Exponent. *The Journal of Developing Areas*, 53(1), 127–145. <https://doi.org/10.1353/jda.2019.0008>