Numerical studies of a high-cycle accumulation model for sand.

Jonas Van Damme
Student number: 01502551

Supervisors: Ir. Dirk Vinckier, Prof. dr. ir. Adam Bezuijen
Counsellors: Dr. ir. Wojciech T Solowski (Aalto University), Dr. ir. Ayman Abed (Aalto University)

Master's dissertation submitted in order to obtain the academic degree of
Master of Science in de industriële wetenschappen: bouwkunde

Academic year 2018-2019
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PREFACE

This dissertation is executed to complete the education `Master of Science in Civil Engineering Technology' at Ghent University. This dissertation has been made during an Erasmus+ exchange program at Aalto University. This written record was made possible by the Departments of Geoengineering and Civil Engineering at both universities.

In this dissertation, the phenomenon of residual strain accumulation in sand soils is treated in a numerical way. This phenomenon was unknown to me at the beginning of my exchange. In this dissertation, this phenomenon is discussed and reproduced.

I had some ups and downs during my work on this dissertation, but at the end I was able to complete it and therefore I would like to acknowledge several people. I would like to thank my counsellors at Aalto University: W. T. Solowski and A. A. Abed for all the feedback during our weekly meetings and for guiding me through this process. I would like to thank Q. Tran and M.J. Seyedan, two PhD students who I shared an office with on campus and who always helped me when I had questions regarding the thesis, but regarding exchange life in general. I would like to thank my two supervisors from Ghent University A. Bezuijen and D. Vinckier for helping arrange the subject of the thesis and for helping me when I had questions regarding the thesis and the exchange.

I would like to thank all the people I’ve met in Finland, who contributed to my semester abroad.

In general, I would like to thank all professors, assistants and researchers from Ghent University and Aalto University, who gave me the possibility to educate myself in a pleasant environment.

Last but not least, I would like to express special thanks to my parents, girlfriend, sister, family, friends and fellow students who supported me during my entire education. It is thanks to them too that I could succeed my entire education.

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Jonas Van Damme, august 2019
Numerical studies of a high-cycle accumulation model for sand

by

Jonas Van Damme

Master's dissertation submitted in order to obtain the academic degree of Master of Science in Civil Engineering Technology

Supervisors: Prof dr. Ir. Adam Bezuijen, Ir. Dirk Vinckier
Counsellors: Prof dr. Ir. Wojciech T. Solowski, Dr. Ir. Ayman A. Abed

Department: Civil Engineering
Chair: Prof. Dr. Ir. Adam Bezuijen
Research group: Geotechnics
Faculty of Engineering and Architecture
Academic year 2018 – 2019
ABSTRACT

This thesis has the aim to understand and reproduce a High-cycle accumulation (HCA) model for sand (Wichtmann, T., Niemunis, A., 2005). This model has great potential, but it requires a vast number of input parameters, based on the results of advanced, laborious laboratory tests. In this thesis, the results of these tests are critically reproduced, using the input parameters from the literature.

The model is used for the prediction of permanent deformation accumulation in granular, non-cohesive soils, caused by long-term cyclic loading with a large number of cycles. This investigation is of importance in many practical cases where the serviceability of a foundation is a main concern. Some examples where such prediction is needed: the on-shore or off-shore wind power plants, railways, watergates, tanks and machine foundation. This thesis focuses only on theoretical cases, not yet for real life practice.

First of all an introduction is given, explaining what will be done in this thesis, what will be discussed in detail (since the scope of the model is wide) and it gives an overview of the different possibilities this model can have and which of them are chosen to reproduce in this thesis.

Thereafter is the High-cycle accumulation model explained in detail, defining every parameter and how they are defined, followed by a description of the way the model works and explaining every equation that is used. Subsequently, the model is reproduced using the software ‘Matlab’.

The material that is the main focus (sand) is explained in detail and the parameters are listed for all the samples that are used. How these parameters are of use and how they are determined is explained.

The model that is reproduced using Matlab, is coded in that way that the different influencing parameters can be compared with the results from the literature. Important parameters, such as the strain amplitude, the functions and the main component: the strain accumulation are focused on. The used assumptions and missing inputs in the literature are listed. The results are discussed and the reason why the results are not perfect is explained.

Keywords: accumulation of strain, cyclic triaxial tests, high-cycle accumulation model, high-cyclic loading, sand
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LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

BVP          Boundary value problem
CDSS        Cyclic Direct Simple Shear
CSL          Critical state line
CSS          Cyclic simple shear
FE           Finite element
FEM          Finite element method
HCA          High-cycle accumulation model
IP           In-phase
OOP          Out-of-phase

Symbols

$c$. [\text{]}] Material parameter
$D_r$ [\text{]}] Relative density
$d_0$ [\text{mm}] Soil particle diameter
$E$ [\text{Pa}] Young’s modulus of elasticity
$E$ [\text{Pa}] Elastic stiffness tensor
$e$ [\text{]}] Void ratio
$e_0$ [\text{]}] Initial void ratio
$e_{\text{max}}$ [\text{]}] Maximum void ratio
$e_{\text{min}}$ [\text{]}] Minimum void ratio
$f.$ [\text{]}] Scalar function in the HCA constitutive model
$g^A$ [\text{]}] Historical cyclic loading ‘memory’
$G$ [\text{Pa}] Shear modulus
$I_{D0}$ [\text{]}] Initial relative density
$K$ [\text{Pa}] Bulk modulus
$M$ [\text{]}] Critical state line from Modified Cam-Clay flow rule
$m$ [\text{]}] Direction of accumulation
$N$ [\text{]}] Number of cycles
$n$ [\text{]}] Porosity
$p$ [\text{N/m}^2] (Roscoe) effective mean pressure
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1 INTRODUCTION

1.1 Objective of this work

An important type of problem in the geo-engineering field rises from cyclic loads (loads with a cyclic recurrence pattern). These types of cyclic loads can be caused by objects which apply periodically variable forces on a soil body. A cyclic loading of a foundation may be caused by recurring peak forces from crossing vehicles, by vibrations caused by the wind, by loading of waves, by changing water levels due to the tide or by rotating unbalances. The cyclic loading can be of a deterministic or a random nature. Also the installation of sheet pile walls or piles using vibratory drivers leads to a cyclic shearing of the surrounding soil. Some densification techniques (deep vibratory compaction, vibratory compaction at the soil surface) make use of cyclic loading in order to improve the mechanical properties of the soil (shear strength, stiffness, liquefaction resistance).

If a foundation passes cyclic loads into a soil, the residual settlement (S) increases with the number of cycles (figure 1). Each cycle causes an irreversible deformation in the soil. The extent of accumulation of residual settlements depends on the loading (average load, load amplitude) of the foundation and on the initial state of the soil. In the case of non-cohesive soils especially the soil density and the fabric of the grain skeleton are of importance. Special attention is given to cyclic loading with sand soil since sand is not a rigid material, since the grains can move with respect to each other. Even small amplitudes can cause significant settlements if the number of cycles is high (for example N > 10³, the so-called poly or high-cyclic loading).

The residual deformations due to cyclic loading concentrate much stronger in the vicinity of the foundation than those due to monotonic loading, leading to larger differential settlements. These differential settlements should be kept within a small range, in order to ensure operational requirements. Thus an accurate prediction of the residual deformations is required for several decades of operation.

This work focuses on the explicit method available (High-cycle accumulation model by Wichtmann), which treats the accumulation of residual strains under cyclic loading similar to the problem of creep under constant loads in viscoplastic models. The case that was simulated is based on cyclic loads on foundations in drained sand. This accumulation model describes the full strain tensor and considers all essential influencing parameters. This model is based on extensive laboratory tests: cyclic triaxial tests and cyclic simple shear (CSS) tests.
1.2 Types of accumulation versus laboratory tests

Accumulation is used in such a way that it can stand for an increase as well as a decrease of the value of a variable. Depending on the boundary conditions, a cyclic loading can lead to residual strain (deformations in a sand body can occur due to compaction, pseudo-creep), change of stress (when boundary conditions restrict straining, pseudo-relaxation) or residual strain and a change of stress (pseudo-creep and pseudo-relaxation simultaneously).

Different types of cyclic laboratory tests are used in the literature, but only the drained cyclic triaxial test (figure 2) results [12] are used as a basis for the high-cycle accumulation model that is reproduced in this thesis. Each type of laboratory test has different types of accumulation depending on the boundary conditions, a quick overview will explain the accumulation for the different types of cyclic triaxial tests.

Other laboratory tests are: simple shear devices and resonant column devices.

The average stress is applied, thereafter the cell pressure is increased and the axial load is increased (compression) or reduced (extension). After a consolidation period (pore pressure is zero) of 1 hour, the average stress was superposed by stress cycles. The axial load is applied with pneumatic loading systems. The first cycle is applied with a frequency of 0.01 Hz, to eliminate unintentional actions caused by the large deformations, all other cycles are applied with a frequency of 1 Hz.

If the cyclic triaxial test is performed in drained conditions, the cell pressure is set to a constant value and the sample is consolidated (pore pressure is zero). The sample is slowly sheared, by applying an axial load (every cycle). The results are the deformations over the whole stress path and the deformations and effective stresses at failure. The drainage is opened during the test (the water can be pushed out of the sample).
If the conditions are undrained then the cell pressure is set to a constant value and the value is consolidated (pore pressure is zero) before shearing. The sample is sheared, but the drainage is closed (water stays in the sample during loading) so the pore pressures increase. The results are the deformations over the whole stress path and the deformations and effective stresses at failure. Undrained tests (short-term analysis) are easier and cheaper to carry out.

The accumulation of strain in the case of (nearly) closed stress loops under cyclic loading conforms with the stress-controlled drained cyclic triaxial test (figure 1 and 3a). In this case a prescribed amount of stress is applied in different cycles, resulting in changing strain values.

If the strain loops are closed, the material reacts with not perfectly closed stress loops and thus the stress accumulates (relaxation). This conforms with the undrained cyclic triaxial test that is controlled by deformation under a constant volume (figure 3b). In this case a prescribed amount of strain is applied in different cycles, resulting in changing stress values.

A simultaneous accumulation of stress and strain is also possible. This can be obtained in an undrained cyclic triaxial test on fully water-saturated specimens with a control of the deviatoric stress (figure 3c). In this case a changing amount of stress is applied in different cycles, resulting in changing strain values.

![Figure 3: Accumulation of stress or strain (one-dimensional case)](image)

In the case of fully water-saturated soils and poor or insufficient drainage conditions, the cyclic shearing can lead to a build-up of excess pore water pressures $u$ (this depends on the soil, it also possible that nothing will happen or suction builds up). These excess pore water pressures cause a reduction of the effective stress $\sigma' = \sigma - u$ and thus a reduction of shear strength. In the special case $\sigma = u$ ($\sigma' = 0$) the soil loses any shear strength and 'liquefies'. This case is of importance for coastal or off-shore structures and cyclic loading caused by seismic shear waves.

The concept of liquefaction is thoroughly studied in literature and is of importance when the number of cycles is low and the strain amplitude is large. This thesis however focuses on the strain accumulation caused by drained cyclic loading when the number of cycles is high and the strain amplitude is small. The stress (load) is a controlled input, this is the case in a drained cyclic triaxial test, because most problems in reality are stress-controlled, which makes it more useful that the model approaches the problem in the same way.
1.3 Type of load

It is important to understand the differences between static, cyclic and dynamic loads, since they require a different calculation approach. The only physical difference however is the time dependency of the load magnitude. The big difference between the types is the reaction of the soil when subjected to those types of loads. The behaviour depends on the time dependency of the loading and the weight, stiffness, strength and composition of the soil. The self-weight of a structure is constant under normal conditions and thus there is no time dependency of the load, this is related to the static failure mechanisms of the soil. When the speed of change of the load magnitude is relatively high (accelerations that are higher than 0.5 m/s²), dynamic effects can be considered. The calculation of the soil accelerations is the main component in a dynamical analysis, since they can cause large fluctuations in stress levels or disrupt the stability of the soil. When the speed of change of the load magnitude is relatively slowly and has a recurring pattern, cyclic effects can be considered. The continuously varying loading of the soil can lead to changes in the soil structure after a certain amount of time. Another type of loading is monotonic loading, examples are tension and compression. This type of loading starts with a static load, which increases in time, until failure occurs. This type of loading is used in many laboratory tests, to determine various properties of soil samples.

This work focuses only on cyclic loading of the soil (figure 1) and monotonic preloading (which is assumed to have no effect on the flow rule or rate of accumulation, if the load cycles occur at the same average stress state).
1.4 Implicit versus Explicit

In Finite Element (FE) calculations of the accumulation due to cyclic loading, two different numerical strategies can be considered: the implicit and the explicit method.

In the implicit procedure each cycle is calculated with a $\sigma - \varepsilon$ constitutive model, the accumulation is a by-product due to the not perfectly closed stress or strain loops, using an elastoplastic or hypoplastic model. The authors of the High-cycle accumulation model used a hypoplastic model with intergranular strain.

The applicability of the implicit method is restricted to a low number of cycles ($N < 50$) because with each increment an accumulation of systematic errors takes place. The large calculation effort also sets boundaries to the application of this implicit method.

The implicit scheme calculates the total strain path (oscillating pattern in grey zones in figure 4), this is a very time consuming approach and can be inaccurate if a lot of cycles are applied. For conventional soil models this is the standard approach.

In the explicit procedure, the total strain is not calculated implicitly in real-time, but the accumulated (plastic) strain is calculated explicitly in 'pseudo-time', which is the number of cycles $N$ replacing the time $t$. This approach is similar to creep under constant loads in viscoplastic models. This approach is less time consuming and is more accurate if a large number of cycles is applied. The explicit method does however need calculations from the implicit method in the first two cycles, to determine the strain amplitude (figure 4).

![Figure 4: Explicit calculation procedure](11)

The strain amplitude is calculated using a hypoplastic model with intergranular strain extension. It is determined from the recording cycle (second cycle), this is not the first cycle, since that cycle causes irregular results and the strain amplitude would be too large. When the amplitude is known, the strain accumulation with increasing number of cycles can be calculated (chapter 2.2). After a certain amount of cycles, the changes in density or stress may have changed the strain amplitude, therefore a recalculation occurs (control-cycle).

This thesis focuses mainly on the explicit numerical strategy, the implicit part (control-cycles) is omitted, although more accurate results can be achieved if both methods are combined.
Explicit accumulation models are developed for the case of an abation (figure 5), this means that the accumulation strain rate decreases with each cycle, but never vanishes completely ($\varepsilon_{acc} \sim \ln(N)$).

![Figure 5: Abation](image)

1.5 Objectives & Methodology

The main objective for this thesis is understanding the High Cycle Accumulation model and performing a numerical study by replicating the output data from the literature, using Matlab software. The model is made by Niemunis A. and Wichtmann T. and it is an explicit accumulation model for non-cohesive soils under cyclic loading. Matlab (matrix laboratory) is a numerical computing environment and programming language developed by MathWorks. Version R2018b (9.5) was used to make the code and a guideline was used to get started [2].

To achieve this main objective, the following actions were proposed:
- Performing a literature study ([5], [6], [11]) to understand the theoretical framework and experience gained with the model. The literature that was mainly focused on in this work is the thesis of Wichtmann T. and a lot of papers of Wichtmann T., Niemunis A. and Triantafyllidis Th.
- Replicating the data from the literature with Matlab R2018b and verifying them.

1.6 Thesis outline

In Chapter 2 the High-Cycle accumulation model is presented and several elements are discussed in detail. The definitions of important parameters that were used are given and the influences of different parameters on the accumulation rate are mentioned. The model is implemented in Matlab and this implementation is discussed thoroughly.

In Chapter 3 the characteristics of the tested material are presented.

Chapter 4 gives an overview of all results that are reproduced using the implemented model in comparison with the results from the literature.

Finally in Chapter 5 the main results of this work are summarized and an outlook on further research options is given.
2 HIGH-CYCLE ACCUMULATION MODEL

2.1 Definitions

To avoid misunderstanding, this chapter provides a clear definition of the quantities that are used in this thesis. The sign convention of mechanics (tension stress and strain (elongation) are positive) is used in the explicit accumulation model instead of the sign convention of soil mechanics (compression stress and compression strain are positive). The reason for this is that most publications on experiments use the sign convention of soil mechanics, whereas in the literature on constitutive modelling the sign convention of mechanics is commonly adopted. This chapter provides the definitions for the case of an axisymmetric loading.

2.1.1 Stress

The effective stress at a point in the three-dimensional space is described by the Cauchy stress tensor \( \sigma \). The axial component is denoted by \( \sigma_1 \) and the lateral one by \( \sigma_2 = \sigma_3 \) (in the triaxial case). The Roscoe invariants \( p \) (mean pressure) and \( q \) (deviatoric stress) are used in the triaxial space (figure 6).

\[
p = \frac{1}{3} (\sigma_{11} + 2 \sigma_{33}) \quad (1)
\]

\[
q = \sigma_{11} - \sigma_{33} \quad (2)
\]

An alternative to \( p \) and \( q \) (equation 1 and 2) are the so-called ‘isomorphic’ variables.

\[
P = \sqrt{3} \, p \quad (3)
\]

\[
Q = \sqrt{\frac{2}{3}} \, q \quad (4)
\]

Using isomorphic variables (equation 3 and 4), two vectors which are orthogonal to each other in the three-dimensional principal stress coordinate system, preserve their orthogonality in the \( P-Q \) -plane. This does not apply to the \( p-q \) -coordinate system.

![Figure 6: Definitions of stress component in the triaxial test](image)
In the p-q-plane the state of stress can be described by the stress ratio:

$$\eta = \frac{q}{p}$$ (5)

or alternatively by $\bar{Y}$:

$$\bar{Y} = -\frac{I_1 I_2}{I_3}$$ (6)

$$Y_c = \frac{9 - \sin^2 \varphi_c}{1 - \sin^2 \varphi_c}$$ (7)

$$\bar{Y}_{av} = \frac{Y - 9}{Y_c - 9}$$ (8)

The function $Y$ of Matsuoka & Nakai is related to $\eta$ as follows:

$$Y = \frac{27(3+\eta)}{(3+2\eta)(3-\eta)}$$ (9)

$$\eta = \frac{3Y-27}{4Y} \pm \sqrt{\left(\frac{3Y-27}{4Y}\right)^2 + \frac{9Y-81}{2Y}}$$ (10)

The $I_i$ in equation 6 are the basic invariants of the stress $\sigma$ (Annex 2).

In equation 7, $\varphi_c$ is the critical friction angle (critical state = progressive deformation without change of stress and volume, figure 7).

Figure 7: Critical state

[18]
The state variable $\bar{Y}$ takes the value 0 for isotropic stresses ($\eta = 0, Y = Y_i = 9$) and 1 for a critical stress ratio ($\eta = M_c(\varphi_c)$ or $\eta = M_e(\varphi_c), Y = Y_e$). The inclinations $M_c$ and $M_e$ of the borderlines in the p-q-plane can be calculated from:

\[
M_c = \frac{6 \sin \varphi}{3 - \sin \varphi}
\]

(11)

\[
M_e = -\frac{6 \sin \varphi}{3 + \sin \varphi}
\]

(12)

Therein $\varphi = \varphi_c$ has to be chosen for the critical state line (CSL, figure 7) and $\varphi = \varphi_p$ for the maximum shear strength ($\varphi_p$ = peak friction angle).

In the triaxial case the stress ratios $K = \sigma_3/\sigma_1$ and $\eta$ (equation 5) are connected via

\[
\eta = \frac{3(1 - K)}{2K + 1}
\]

(13)

For $K = 0.5$ one obtains $\eta = 0.75$ and $\bar{Y} = 0.341$.

Figure 8 shows a typical stress-path in p-q-plane for a cyclic triaxial test. An average stress $\sigma^{av}$ (described by $p^{av}$ and $q^{av}$ or $\eta^{av}$ or $\bar{Y}^{av}$) is superposed by a cyclic portion. An oscillation of the axial and the lateral stresses $\sigma_1(t)$ and $\sigma_3(t)$ without a phase-shift in time (in-phase cycles, see Section 2.1.4) results in stress cycles along a straight line with a certain inclination $\tan \alpha = q^{ampl}/p^{ampl}$ in the p-q-plane. For the special case of constant lateral stresses ($\sigma_3^{ampl} = 0$) $\tan \alpha = 3$ holds and the amplitude ratio

\[
\zeta = \frac{\sigma_1^{ampl}}{p^{av}} = \frac{q^{ampl}}{p^{av}}
\]

(14)

is used.
2.1.2 Strain

The definitions are explained for the strain $\varepsilon$, but they are also valid for the strain rate $\dot{\varepsilon}$.

Strain and strain rate are used in this thesis, in the context of cyclic loading 'rate' means a derivative with respect to the number of cycles $N$ ($\varepsilon = \partial x / \partial N$) instead of time $t$ (in which the discrete number of cycles $N$ is treated as a ‘smoothed’ continuous variable).

Consider a rigid box filled with sand, the height of the sand in the box is $L$. When a uniform vertical stress $\sigma$ is applied to the surface, the sand will deform with a vertical deformation $u$ (figure 9). The vertical deformation can be expressed as a (dimensionless) strain $\varepsilon$, by dividing the vertical deformation by the original height of the sample:

$$\varepsilon = \frac{u}{L} \quad (15)$$

![Figure 9: Sand subjected to a uniform vertical pressure](image)

The amount of strain caused by the applied stress will depend on the magnitude of the stress and the stiffness properties of the sand. It can be assumed that the applied stress is relatively small and thus the deformation that is displayed in figure 9 is exaggerated, in reality the deformations will almost not be visible to the human eye. Sand is also considered to behave like a linear elastic material (see section 2.2.1), if the stresses and deformations are small.

In this thesis, the soil sample is not a rigid box, but a cylinder, the strain components in this case are shown in figure 10.

![Figure 10: Definition of strain components in the triaxial test](image)

The strain in the axial direction is denoted with $\varepsilon_1$ and the strain in the lateral direction with $\varepsilon_2 = \varepsilon_3$ (figure 10). The strain invariants

$$\varepsilon_v = \varepsilon_{11} + 2\varepsilon_{33} \quad (16)$$

$$\varepsilon_q = \frac{2}{3}(\varepsilon_{11} - \varepsilon_{33}) \quad (17)$$

are used. The rates of volumetric strain $\dot{\varepsilon}_v$ and the deviatoric strain $\dot{\varepsilon}_q$ are work-conjugated to the Roscoe invariants $p$ and $q$. 
The total strain is
\[ \varepsilon = \sqrt{(\varepsilon_{11})^2 + 2(\varepsilon_{33})^2} \] (18)

As an alternative to \( \varepsilon_q \), the shear strain
\[ \gamma = \varepsilon_{11} - \varepsilon_{33} \] (19)
can be used.

If the applied vertical stress is increased, it is possible to think that the vertical strain will also increase. If thereafter the stress is reduced to the previous stress state, it is also imaginable that the strains return to the previous state, since it was assumed that the sand will 'bounce back' elastically. What this model includes is that in reality, a small irreversible deformation has occurred by applying the stress, which prevents the sand from 'bouncing back' to the original state. The actual process in real life is complex and not completely understood, it is possible that some grains undergo a translation or a rotation with respect to each other.

This increase and decrease of stress state is defined as a load cycle with an amplitude \( \sigma_{\text{ampl}} \). The vertical strain resulting from each load cycle can be decomposed in a cyclic (elastic, resilient) oscillating part \( \varepsilon^{e}(\text{or } \varepsilon^{r}) \) with \( \varepsilon_{\text{ampl}} \) and an accumulated (plastic) part \( \varepsilon^{\text{acc}} \) (trend) (figure 11 and 12).

![Figure 11: Evolution of strain during application of stress](image1)

![Figure 12: Evolution of total strain \( \varepsilon \) in a cyclic triaxial test](image2)

The fundamental idea of strain accumulation is that a load cycle with a (relatively) small amplitude leads to very small irreversible strains. This effect is only noticeable over a large number of cycles, figure 11.
and 12 are thus showing an exaggerated image of this theory. This model only considers 1 component of the stress state, being the vertical normal stress (1D). In a realistic situation (3D) the cyclic loading has an effect on all six independent components of the stress state.

2.1.3 Pore volume

The magnitude of pore volume is described by the void ratio $e$ or the porosity $n$. The density index is calculated from the maximum ($e_{\text{max}}$) and minimum ($e_{\text{min}}$) void ratios as follows:

$$I_D = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}}$$

(20)

The initial value of the density index at the beginning of a test is denoted by $I_D^0$, also the relative density $D_r$ is often used in the literature. This is explained more in detail in chapter 3.

2.1.4 Shape of the cycles

There is a distinction between the so-called in-phase (IP) and out-of-phase (OOP) – cycles.

In this thesis, only the uniaxial in-phase (IP) cycles (figure 13) are considered. All components of $\varepsilon$ oscillate with the same scalar, periodic function $-1 \leq f(t) \geq 1$ in the time $t$, these cycles are addressed as ‘one-dimensional’. In the cyclic triaxial test with $\sigma_3 = \text{constant}$, only the axial component $\sigma_1$ varies with time, so the IP-cycles are uniaxial:

$$\varepsilon = \varepsilon^{\text{av}} + \begin{pmatrix} \varepsilon_{1}^{\text{ampl}} \\ 0 \\ 0 \end{pmatrix} f(t)$$

(21)

![Figure 13: Uniaxial IP-cycles](image)

Figure 13: Uniaxial IP-cycles [11]
2.2 Description

The High-cycle accumulation model is one of the most recent and advanced accumulation models available. It is able to calculate the accumulation of stresses and strains in sand, which is subjected to cyclic loading with a small amplitude and a high number of cycles. The model has been validated with numerous series of cyclic triaxial tests, where up to 2 million load cycles were applied. This HCA model requires the amplitude of cyclic straining ($\varepsilon^{\text{ampi}}$) as an input parameter. This main parameter is calculated numerically with another soil model, since it’s difficult to acquire it accurately from in-situ measurements.

In this chapter, the constitutive equation is explained (2.2.1) and the calculation procedure of the intensity (2.2.2) and direction (2.2.3) of accumulation is clarified. This model has an empirical character, since the explicit formulations are formed by fitting mathematical expressions to laboratory test results.

2.2.1 Constitutive equation

The basic constitutive equation [7] reads

$$\dot{\sigma} = E \cdot (\dot{\varepsilon} - \dot{\varepsilon}^{\text{acc}} - \dot{\varepsilon}^{\text{pl}})$$  \hspace{1cm} (22)

The equation can also be expressed in matrix formation (2D)

$$\begin{bmatrix} \dot{\sigma}_{11} \\ \dot{\sigma}_{22} \\ \dot{\sigma}_{33} \\ \dot{\sigma}_{12} \\ \dot{\sigma}_{13} \\ \dot{\sigma}_{23} \end{bmatrix} = \frac{E}{(1 + v)(1 - 2v)} \begin{bmatrix} 1 - v & v & 0 & 0 & 0 \\ v & 1 - v & v & 0 & 0 \\ v & v & 1 - v & 0 & 0 \\ 0 & 0 & 0 & 1 - 2v & 0 \\ 0 & 0 & 0 & 0 & 1 - 2v \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_{11} - \dot{\varepsilon}_{11}^{\text{acc}} - \dot{\varepsilon}_{11}^{\text{pl}} \\ \dot{\varepsilon}_{22} - \dot{\varepsilon}_{22}^{\text{acc}} - \dot{\varepsilon}_{22}^{\text{pl}} \\ \dot{\varepsilon}_{33} - \dot{\varepsilon}_{33}^{\text{acc}} - \dot{\varepsilon}_{33}^{\text{pl}} \\ \dot{\varepsilon}_{12} - \dot{\varepsilon}_{12}^{\text{acc}} - \dot{\varepsilon}_{12}^{\text{pl}} \\ \dot{\varepsilon}_{13} - \dot{\varepsilon}_{13}^{\text{acc}} - \dot{\varepsilon}_{13}^{\text{pl}} \\ \dot{\varepsilon}_{23} - \dot{\varepsilon}_{23}^{\text{acc}} - \dot{\varepsilon}_{23}^{\text{pl}} \end{bmatrix}$$  \hspace{1cm} (23)

The constitutive equation can also be rewritten with Roscoe’s invariants, for axisymmetric element tests

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = [K \ 0 \ 0 \ 3G] \begin{bmatrix} \dot{\varepsilon}_v - \dot{\varepsilon}_v^{\text{acc}} - \dot{\varepsilon}_v^{\text{pl}} \\ \dot{\varepsilon}_q - \dot{\varepsilon}_q^{\text{acc}} - \dot{\varepsilon}_q^{\text{pl}} \end{bmatrix}$$  \hspace{1cm} (24)

The following components are part of the equations:

- $\dot{\sigma}$ = stress rate of the effective stress $\sigma$ [Pa]
- $E$ = barotropic stress-dependent elastic stiffness tensor [Pa]
- $\dot{\varepsilon}$ = total strain rate [/]
- $\dot{\varepsilon}^{\text{acc}}$ = accumulated strain rate [/]
The bulk modulus is obtained experimentally from a comparison of the rate of pore pressure accumulation ($\dot{u}$) in an undrained cyclic triaxial test and the rate of volumetric strain accumulation ($\dot{\varepsilon}_v$) in a drained cyclic test, with similar initial stress, similar initial void ratio and with the same cyclic loading ($q^{ampl}$ should be the same). The bulk modulus is the material's response to hydrostatic pressure [17].

$$K = \frac{\dot{u}}{\dot{\varepsilon}_v}$$  \hspace{1cm} (25)

In order to evaluate $\dot{u}$ and $\dot{\varepsilon}_v^{acc}$ for exactly the same test conditions (same values for $\varepsilon^{ampl}$, $e^{av}$ and $g^k$), the rate from the drained test $\dot{\varepsilon}_v^{acc}$ should be corrected by a factor

$$(f_{ampl}^U/f_{ampl}^D)(f_{p}^U/f_{p}^D)(f_{\dot{u}}^U/f_{\dot{u}}^D)(f_{\dot{\varepsilon}_v}^{acc} U/f_{\dot{\varepsilon}_v}^{acc} D),$$

with $^U$ and $^D$ indicating the undrained and drained test, respectively. To be able to determine the barotropy of $K$, at least two pairs of tests with different pressures should be carried out, it was found that the bulk modulus can be approximated by the following equation [17] :

$$K = A \ p_{atm}^{n-1} \ (p^{av})^n$$  \hspace{1cm} (26)

With:

$$A = 467 \ \ \ [17]$$
$$n = 0.46 \ \ \ [17]$$
$$p_{atm} = p_{ref} = 100.000 \ \ \ [kPa]$$

Since we are assuming in this case that the sand is a transversely isotropic material, the following relations regarding the Bulk modulus and Young’s modulus and Shear modulus are valid.

- $E = $ Young’s modulus \hspace{1cm} [Pa]

The Young’s modulus (input for the Hooke stiffness tensor $E$) is calculated using the following conversion formula, based on the bulk modulus

$$E = 3K(1 - 2\nu)$$  \hspace{1cm} (27)

The Young’s modulus describes the response of the soil’s strain to uniaxial stress in the direction of the stress. The stiffness is isotropic [17].
• \( G = \text{Shear modulus} \) \hspace{1cm} \text{[Pa]} \\
\[
G = \frac{E}{2(1 + \nu)} \tag{28}
\]

The shear modulus can also be calculated based on the bulk modulus, instead of the elastic stiffness:

\[
G = \frac{3K \ast (1 - 2 \ast \nu)}{2 \ast (1 + \nu)} \tag{29}
\]

The shear modulus (rigidity) is the ratio of shear stress to shear strain.

• \( \dot{\varepsilon}^{pl} = \text{plastic strain rate (for stress paths touching the yield surface)} \) \hspace{1cm} [1]

The plastic strain rate \( \dot{\varepsilon}^{pl} \) is caused by monotonic loading (yield condition of Matsuoka and Nakai is used), but physically it isn’t a different phenomenon then the accumulated strain rate \( \dot{\varepsilon}^{acc} \). The two parts are separated because it is necessary for the explicit formulation. The requirement of the plastic strain rate can best be explained by the effect of neglecting it. If it is neglected, there can be situations where neighbouring elements have a different strain accumulation, leading to contractions that need to be compensated by the dilation of other elements (conservation of total area). To prevent the stress state from crossing the Coulomb failure surface, the plastic strain rate is inserted, leading to a stress state on the yield surface. In this thesis, the plastic portion is neglected \( (\dot{\varepsilon}^{pl} = 0) \) for now (it is easy to include in future FE calculations).

• \( \nu = \text{Poisson’s ratio} \) \hspace{1cm} [1]

Poisson’s ratio is the negative of the ratio of transverse strain to axial strain, it measures the effect where a material tends to expand in directions perpendicular to the direction of compression. If the boundary conditions allow compression/extension in more than direction, the Poisson’s ratio can be determined as follows:

\[
\nu = \frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \frac{-\Delta L’/L’}{\Delta L/L} \tag{30}
\]

Figure 14 shows the physical meaning of the Poisson’s ratio and the elements that are used in equation 30.

\begin{center}
\textit{Figure 14: Determination of poisson’s ratio [10]}
\end{center}
2.2.2 Intensity of accumulation

The rate of strain accumulation $\dot{\varepsilon}^{\text{acc}}$ is calculated as a product of the scalar intensity of accumulation $\dot{\varepsilon}^{\text{acc}}$ and the tensorial direction of accumulation $m$ ("flow rule"), the value is recalculated in each cycle according to

$$\dot{\varepsilon}^{\text{acc}} = \dot{\varepsilon}^{\text{acc}} m = f_{\text{ampl}} \dot{\varepsilon}_N f_e f_p f_y f_n m$$  \hspace{1cm} (31)

In this formula, $m$ accounts for the direction of accumulation, the factors $f_{\text{ampl}}, \dot{\varepsilon}_N, f_e, f_p, f_y$ and $f_n$ account for the intensity of accumulation.

The intensity of accumulation is composed of six multiplicative functions, each with the following influencing parameters.

- $f_{\text{ampl}}$: strain amplitude $\varepsilon^{\text{ampl}}$

The influence of the shape of the strain loop on the accumulation rate is captured by a tensorial definition of the strain amplitude $A_\varepsilon$ (4-th order tensor). The scalar measure $\varepsilon^{\text{ampl}}$ is the Euclidean norm of the tensorial strain amplitude $A_\varepsilon$.

$$\varepsilon^{\text{ampl}} = \| A_\varepsilon \|$$  \hspace{1cm} (32)

For one-dimensional (in-phase) cycles, which this thesis focuses on, the strain amplitude can also be calculated with the classical definition of the amplitude:

$$\varepsilon^{\text{ampl}} = \frac{\varepsilon^{\max} - \varepsilon^{\min}}{2}$$  \hspace{1cm} (33)

For two-dimensional cases, the strain amplitude is calculated implicitly.

The magnitude of the strain amplitude $\varepsilon^{\text{ampl}}$ (scalar) is an essential control parameter for $\dot{\varepsilon}^{\text{acc}}$. A linear relationship between the accumulation rate and a second power law of the strain amplitude exists, therefore $f_{\text{ampl}}$ is calculated with

$$f_{\text{ampl}} = \min \left\{ \left( \frac{\varepsilon^{\text{ampl}}}{\varepsilon^{\text{ref}}} \right)^2 ; 100 \right\}, \quad \varepsilon^{\text{ref}} = 10^{-4}$$  \hspace{1cm} (34)

This equation is only valid if $5.10^{-5} < \varepsilon^{\text{ampl}} < 5.10^{-3}$, because this mechanism only works for (relatively) small strain cycles. If the cyclic strain amplitude would become too large, each load cycle would deform the grain structure, leading to an alternative deformation mechanism and thus another approach of calculation is needed. For strain amplitudes larger than $10^{-3}$, the accumulation rate is almost independent of $\varepsilon^{\text{ampl}}$. The strain amplitude is defined in chapter 1.4 and calculated in chapter 2.3. This function is visualised for different values of the strain amplitude in figure 30.
• \( \dot{f}_N \): historiotropy (cyclic preloading) scalar \( g^A \), number of cycles \( N \)

This function includes the influence of the number of cycles \( N \), the rate of accumulation increases with \( N \) proportionally to

\[
\dot{f}_N = C_{N1} C_{N2} \exp \left[ - \frac{g^A}{C_{N1} f_{\text{ampl}}} \right] + C_{N1} C_{N3}
\]

\[
\dot{f}_N = \dot{f}_N^B + \dot{f}_N^R
\]  

(35)

This function has a constant portion:

\[
\dot{f}_N^B = C_{N1} C_{N3}
\]  

(36)

and a \( N \)-depending portion:

\[
\dot{f}_N^A = C_{N1} C_{N2} \exp \left[ - \frac{g^A}{C_{N1} f_{\text{ampl}}} \right]
\]  

(37)

However, \( N \) is not the only variable whereof the cyclic preloading is depending, to include the information about the intensity of the cycles in the past, the preloading variable \( g^A \) is introduced:

\[
g^A = \int f_{\text{ampl}} \frac{C_{N1} C_{N2}}{1 + C_{N2} N}
\]  

(38)

This variable counts the cycles by weighting them with their amplitude. Only the \( N \)-depending portion of \( \dot{f}_N \) is considered for \( g^A \).

The cyclic preloading has a big influence to the magnitude of \( \dot{\varepsilon}^{\text{acc}} \). Different samples with the same void ratio and who are subjected to the same average stress, but with a different history of (cyclic) loading have significantly different rates of strain accumulation (figure 15a). Not only the number of these cycles is of importance, but also the amplitudes have an influence on the current densification rate \( \dot{\varepsilon} = \frac{d \varepsilon}{d N} \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.png}
\caption{a) Effect of the cyclic history on the rate of densification, b) Determination of \( C_{N1}, C_{N2} \) and \( C_{N3} \) \cite{18}}
\end{figure}

For a constant amplitude, the HCA model predicts accumulation curves proportional to a logarithmic and a linear portion:

\[
f_N = C_{N1} \left[ \ln (1 + C_{N2} N) + C_{N3} N \right]
\]  

(39)
Parameters $C_{N1}$, $C_{N2}$ and $C_{N3}$:

The parameters $C_{N1}$, $C_{N2}$ and $C_{N3}$ are determined using the same data from where $C_e$, $C_p$ and $C_Y$ are determined. To eliminate the influence of the strain amplitude, the void ratio, the average mean pressure and the average stress, the accumulated strain is divided by $f_{amp} f_e f_p f_Y$. For each number of cycles $N$ a mean value of the strain amplitude is calculated, as well as a mean value of the void ratio. $\frac{\varepsilon_{acc}}{f_{amp} f_e f_p f_Y}$ is plotted (figure 15b) versus $N$ and equation 35 has to be fitted to this data, so that $C_{N1}$, $C_{N2}$ and $C_{N3}$ are known. This is done for every cycle, but only the average value is necessary for the model.

- $f_e$: void ratio $e$

Loose sands will compact more easily than dense sands, the influence of the void ratio on the strain accumulation is expressed by the following hyperbolic function

$$
\frac{\varepsilon_{acc}}{f_{amp} f_e f_p f_Y} = \frac{(C_e - e)^2}{1 + e_{ref} (C_e - e_{ref})^2}, \quad e_{ref} = e_{max}
$$

(40)

This function is based on the test results where the initial void ratios are changed, but the stresses are identical. The constant stress amplitude $q_{amp}$ leads to an increase of the accumulated strain rates with increasing void ratio (figure 16a). For up to 200 cycles there is an increase of $C_e$ with $N$, for larger values is $C_e$ almost constant.

![Figure 16: a) Intensity of accumulation at different initial void ratios, b) Determination of $C_e$](image)

Parameter $C_e$:

The parameter $C_e$ (asymptotic void ratio) corresponds to the void ratio for which the accumulation rate vanishes ($f_e = 0$), that means for which $\dot{\varepsilon}_{acc} = 0$ holds. $C_e$ is related to the minimum void ratio $e_{min}$, $e$ is the current void ratio and the maximum void ratio is $e_{max}$. To calculate the parameter $C_e$, a series (at least 3) of load-controlled drained cyclic triaxial tests needs to be executed with different initial densities while identical average stresses and stress amplitudes are used throughout the series (figure 16b). To eliminate the influence of the strain amplitude, the accumulated strain is divided by $f_{amp}$. For each number of cycles $N$ a mean value of the strain amplitude is calculated, as well as a mean value of the void ratio. The ratio $\varepsilon_{acc}/f_{amp}$ is plotted versus $e$ and equation 40 has to be fitted to this data, so that
$C_e$ is known. This is done for every cycle, but only the average value is necessary for the model. For up to 10000 cycles there is an increase of $C_e$ with $N$, for larger values is $C_e$ (almost) constant.

- $f_p$: average mean pressure $p^{av}$

The accumulation rate decreases exponentially with $p^{av}$ according to the following equation

$$f_p = \exp\left[-C_p \left(\frac{p^{av}}{p^{ref}} - 1\right)\right], \quad p^{ref} = p^{atm} = 100 \text{ kPa} \quad (41)$$

This function is based on the test results where the average mean pressure is varied between 50 and 300 kPa, but the stresses are identical. The constant stress amplitude $q^{ampl}$ leads to an increase of the accumulated strain rates with increasing average mean pressure (figure 17a).

![Figure 17: a) Intensity of accumulation at different average mean pressures, b) Determination of $C_p$](image)

Parameter $C_p$:

To calculate the parameter $C_p$, a series of load-controlled drained cyclic triaxial tests needs to be executed with a variation of the average mean pressure $p^{av}$ while keeping the average stress ratio $\eta^{av}$, the amplitude-pressure ratio $\zeta = q^{ampl}/p^{av}$ and the initial density constant. The amplitude-pressure ratio is chosen with respect to the boundary value problem (BVP), so that the accumulation of strain does not become excessive under some test conditions. To eliminate the influence of the strain amplitude and the void ratio, the accumulated strain is divided by $f_{ampl}$ and $f_e$. For each number of cycles $N$ a mean value of the strain amplitude is calculated, as well as a mean value of the void ratio. The ratio $\frac{\varepsilon^{acc}}{f_{ampl}f_e}$ is plotted versus $p^{av}$ (figure 17b) and equation 41 has to be fitted to this data, so that $C_p$ is known. This is done for every cycle, but only the average value is necessary for the model. For up to 200 cycles there is an increase of $C_e$ with $N$, for larger values is $C_e$ almost constant.
• \( f_Y \): average stress ratio \( \eta_{av} \) or \( Y_{av} \)

The accumulation rate increases exponentially with \( Y_{av} \) according to the following equation (figure 18b)

\[
f_Y = \exp[C_Y Y_{av}]
\]

\[
Y_{av} = \frac{Y - 9}{Y_c - 9}
\]  \hspace{1cm} (8)

\[
Y_c = \frac{9 - \sin^2 \varphi_c}{1 - \sin^2 \varphi_c}
\]  \hspace{1cm} (7)

\[
Y = \frac{27(3 + \eta)}{(3 + 2\eta)(3 - \eta)}
\]  \hspace{1cm} (9)

The accumulation rate increases with increasing average stress ratio (figure 18a).

Parameter \( C_Y \):

To calculate the parameter \( C_Y \), a series of load-controlled drained cyclic triaxial tests needs to be executed with different values of \( \eta^\text{av} \) at \( p^\text{av} = \) constant, with \( q^\text{amp} = \) constant and with similar densities (figure ). To eliminate the influence of the strain amplitude and the void ratio, the accumulated strain is divided by \( f_{\text{amp},0} \) and \( f_c \). For each number of cycles \( N \) a mean value of the strain amplitude is calculated, as well as a mean value of the void ratio. The ratio \( \epsilon_{\text{acc},f} / f_{\text{amp},0} \) is plotted versus \( Y_{av} \) and equation 42 has to be fitted to the \( \epsilon_{\text{acc}} / f_{\text{amp},0} - Y_{av} \) data, so that \( C_Y \) is known. This is done for every cycle, but only the average value is necessary for the model. For up to 200 cycles there is an increase of \( C_e \) with \( N \), for larger values is \( C_e \) almost constant.
• \( f_\pi \): polarization changes

The polarization of the strain amplitude is calculated as followed

\[
\overline{A}_\varepsilon = \frac{A_\varepsilon}{\|A_\varepsilon\|}
\] (43)

The polarization during the previous cycles is memorized in the so-called 'back polarization' tensor \( \pi \). In the case of a change of the polarization, the accumulation rate is increased by the following function (for two-dimensional cases)

\[
f_\pi = 1 + C_\pi(1 - \cos \alpha)
\] (44)

This function depends on the angle \( \alpha \), which is included by the actual polarization \( A_\varepsilon \) and the 'back polarization' \( \pi \).

\[
\cos \alpha = \overline{A}_\varepsilon \lrcorner \pi
\] (45)

Only one-dimensional tests have been performed, this means that the accumulation rate is not increased due to the change from package “a” to a subsequent package “b” and thus \( f_\pi = 1 \).

What can be concluded from the equations that are part of the intensity of strain accumulation is that the direction of accumulation depends on a lot of parameters. These parameters are: strain amplitude \( \varepsilon_{\text{ampl}} \), number of cycles \( N \), cyclic preloading \( g^A \), void ratio \( e \), average mean pressure \( p^{av} \) and average stress ratio \( \eta^{av} \). The influence of the span, shape, polarization and polarization changes of the loops is neglected in this thesis. The loading frequency has no effect on the intensity of accumulation. All parameters are dependent on the grain size distribution (figure 24) that is used and are listed in chapter 3.
2.2.3 Direction of accumulation

- **m**: average stress ratio $\eta_{av}$ and critical friction angle $\varphi_c$

The accumulation $\dot{\varepsilon}_{acc}$ has both a volumetric and a deviatoric portion, the ratio is known as the direction of accumulation. The direction mainly depends on the average stress ratio $\eta$. The modified Cam Clay (MCC) cyclic flow rule is used to approximate the direction of accumulation

$$m = \left[ \frac{1}{3} \left( p - \frac{q^2}{M^2 p} \right) 1 + \frac{3}{M^2} \sigma \right]^{-\sigma}$$ \hspace{1cm} (46)

For triaxial extension ($\eta = q/p < 0$) a small modification $M$ (critical state line) is used with

$$M_c = \frac{6 \sin \varphi_c}{3 - \sin \varphi_c}$$ \hspace{1cm} (11)

For triaxial compression ($\eta = q/p > 0$) a small modification $M$ is used with

$$M_e = -\frac{6 \sin \varphi_c}{3 + \sin \varphi_c}$$ \hspace{1cm} (12)

Wherein $\varphi_c$ is the critical friction angle.

$$F = \begin{cases} 
1 + \frac{M_e}{3} \eta & \text{for } \eta \leq M_e \\
1 + \frac{M_e}{3} \eta & \text{for } M_e < \eta < 0 \\
1 & \text{for } \eta \geq 0
\end{cases}$$ \hspace{1cm} (47)

$$M = F M_c$$

$$= F M_e$$ \hspace{1cm} (48)

Equation 46 in Matrix notation is as follows:

$$\begin{bmatrix} m_{11} \\ m_{22} \\ m_{33} \\ m_{12} \\ m_{13} \\ m_{23} \end{bmatrix} = \sqrt{\frac{1}{6} \left( m_{11}^2 + m_{22}^2 + m_{33}^2 + m_{12}^2 + m_{13}^2 + m_{23}^2 \right)}$$ \hspace{1cm} (49)

Equation 31 in Matrix notation is as follows:

$$\begin{bmatrix} \dot{\varepsilon}_{acc}^{11} \\ \dot{\varepsilon}_{acc}^{12} \\ \dot{\varepsilon}_{acc}^{13} \\ \dot{\varepsilon}_{acc}^{22} \\ \dot{\varepsilon}_{acc}^{23} \\ \dot{\varepsilon}_{acc}^{33} \end{bmatrix} = \dot{\varepsilon}_{acc}^{acc}$$ \hspace{1cm} (50)
As can be seen in the equations (34 - 42), the direction of accumulation mainly depends on the average stress ratio and is also influenced by the critical friction angle. It is not influenced by

- Average mean pressure $p^\text{av}$

Figure 19 shows the ratio of the volumetric and the deviatoric portion of the accumulated strain for different average mean pressures. The results show little scatter and are all on one line, meaning that they don’t influence the direction.

![Figure 19: Direction of accumulation for different average mean pressures](image)

- Void ratio $e$

Figure 18 shows the ratio of the volumetric and the deviatoric portion of the accumulated strain for different initial void ratios. The results show little scatter and are all on one line, meaning that they don’t influence the direction.

![Figure 20: Direction of accumulation for different initial densities](image)
• Loading frequency $f_B$

Figure 21 shows the ratio of the volumetric and the deviatoric portion of the accumulated strain for different loading frequencies. The results show little scatter and are all on one line, meaning that they don’t influence the direction.

![Figure 21: Direction of accumulation for different loading frequencies](image)

• Grain size distribution

Figure 22 shows the ratio of the volumetric and the deviatoric portion of the accumulated strain for different grain size distributions. The results show a bit of scatter, but this is because of the difference in strain accumulation rates.

![Figure 22: Direction of accumulation for different grain size distributions](image)
2.3 Model reproduction in Matlab

In this chapter, the Matlab code (available at [https://github.ugent.be/jonavdam/Thesis](https://github.ugent.be/jonavdam/Thesis)) is explained more in detail, since it was not straightforward to get all the input parameters from the literature [11-17]. When reading this chapter, every calculation step should be understandable.

The paper that was mainly focused on is: ‘validation and calibration of a high-cycle accumulation model based on cyclic triaxial tests on eight sands by Wichtmann et al.’ [15].

The code was tested for every sand (figure 24) that was mentioned in that paper and for different numbers of cycles, but this chapter will mainly focus on sand number 3 and a maximum of 100,000 cycles.

The core of this model is the following equation:

\[
\dot{\varepsilon}_{\text{acc}} = \dot{\varepsilon}_{\text{acc}} m = f_{\text{amp}l} \dot{f}_N f_p f_y f_p m
\]  

(31)

First the functions of the intensity of accumulation are clarified:

- \(f_{\text{amp}l}\):

\[
f_{\text{amp}l} = \min\left(\left(\frac{\varepsilon_{\text{amp}l}}{\varepsilon_{\text{ref} \text{amp}l}}\right)^2; 100\right), \quad \varepsilon_{\text{ref} \text{amp}l} = 10^{-4}
\]  

(34)

The strain amplitude is calculated using the constitutive equation (equation 24) from the model, but with a constant elastic strain (the plastic and accumulated elastic parts are omitted):

\[
\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{bmatrix} \varepsilon_v \\ \varepsilon_q \end{bmatrix}
\]  

(51)

\[
\begin{align*}
\Rightarrow [p_{\text{amp}l} \quad q_{\text{amp}l}] &= \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{bmatrix} \varepsilon_{v \text{el}} \\ \varepsilon_{q \text{el}} \end{bmatrix} \\
\Leftrightarrow \begin{cases} p_{\text{amp}l} = K \varepsilon_{v \text{el}} \\ q_{\text{amp}l} = 3G \varepsilon_{q \text{el}} \end{cases} \\
\Rightarrow \begin{cases} \frac{p_{\text{amp}l}}{K} = \varepsilon_{v \text{el}} \\ \frac{q_{\text{amp}l}}{3G} = \varepsilon_{q \text{el}} \end{cases}
\end{align*}
\]

The code is based on the cyclic triaxial test: \(p_{\text{amp}l} = \frac{q_{\text{amp}l}}{3}\), if it would be based on the cyclic direct simple shear test [9] : \(p_{\text{amp}l} = 0\).

\[
\begin{align*}
\Rightarrow \begin{cases} \frac{q_{\text{amp}l}}{3K} = \varepsilon_{v \text{el}} \\ \frac{q_{\text{amp}l}}{3G} = \varepsilon_{q \text{el}} \end{cases}
\end{align*}
\]
\[ K = A \ p_{atm}^{n-1} \ (p^{av})^n \]  

(26)

In the literature from 2009, the value for \( A = 467 \) (the same for all sands). In the first phase, this was also the value that was used in the code. But because the results were not completely similar, this value was changed (not constant for all sands) by curve-fitting the curves (using scale factor \( S^{av} \)) from the code to the curves from the literature (See section 3). The average value of \( A \) for sand no. 3 is 919.8245965 (when \( q^{amp} = 80 \) kPa).

\[ A^{av} = \frac{467}{\sqrt{S^{av}}} \]  

(52)

\( n = 0.46 \)

\[ G = \frac{3K \ (1 - 2 \ \nu)}{2 \ (1 + \nu)} \]  

(29)

Poisson’s ratio \( \nu = 0.2 \), this value is also not straightforward in the literature, but 0.2 is the value that is used in most of the cases.

The atmospheric pressure \( p_{atm} = 100.000 \) Pa.

The average pressure \( p^{av} = 200.000 \) Pa.

\[ \varepsilon^{el} = \varepsilon_{11}^{el} + 2 \ \varepsilon_{33}^{el} \]  

(16)

\[ \varepsilon_{q}^{el} = \frac{2}{3} (\varepsilon_{11}^{el} - \varepsilon_{33}^{el}) \]  

(17)

\[ \Rightarrow \varepsilon_{11}^{el} = \frac{\varepsilon_{q}^{el} + \varepsilon_{11}^{el}}{3} \]

\[ \Rightarrow \varepsilon_{33}^{el} = \frac{\varepsilon_{q}^{el} - \varepsilon_{11}^{el}}{2} \]

\[ \Rightarrow \varepsilon^{el} = \sqrt{(\varepsilon_{11}^{el})^2 + (\varepsilon_{33}^{el})^2} \]

\[ \Rightarrow \varepsilon^{amp} = \varepsilon^{el} \]

\[ \hat{f}_N : \]

\[ \hat{f}_N = f_N^A + f_N^B \]  

(35)

\[ f_N^B = C_{N1} \ C_{N3} \]  

(36)

\[ f_N^A = C_{N1} \ C_{N2} \ \exp \left[ -\frac{g^{A}}{C_{N1} \ f_{amp}} \right] \]  

(37)

For sand number 3 the parameters are: \( C_{N1} = 0.00036 \), \( C_{N2} = 0.43 \) & \( C_{N3} = 0.00005 \).
The preloading variable $g^A$ was modelled as follows:

$$g^A = \int f_{\text{ampl}} \frac{C_{N1} C_{N2}}{1 + C_{N2} N}$$

(38)

For $N = 1$:

$$g^A(N) = f_{\text{ampl}}(N) \frac{C_{N1} C_{N2}}{1 + C_{N2} N}$$

(53)

For $N > 1$:

$$g^A(N) = g^A(N - 1) + \left( f_{\text{ampl}}(N - 1) \frac{C_{N1} C_{N2}}{1 + C_{N2} (N-1)} \right) + \left( f_{\text{ampl}}(N - 1) \frac{C_{N1} C_{N2}}{1 + C_{N2} N} \right)$$

(54)

- $f_e$:

$$f_e = \frac{(C_e - e)^2}{1 + e} \left( 1 + e_{\text{ref}} \right), \quad e_{\text{ref}} = e_{\text{max}}$$

(40)

For sand number 3 the parameters are: $C_e = 0.54$, $e_{\text{ref}} = 0.874$, $e_{\text{min}} = 0.577$

The void ratio is calculated as follows:

$$e = e_{\text{max}} - l_{\text{Di}} (e_{\text{max}} - e_{\text{min}})$$

(55)

With $l_{\text{Di}}$ being the initial relative density, estimated using the curves from the paper, since the exact value was not given, being 0.60 for sand 3 (table 7).

- $f_p$:

$$f_p = \exp \left[ -C_p \left( \frac{p_{\text{av}}}{p_{\text{ref}}} - 1 \right) \right], \quad p_{\text{ref}} = p_{\text{atm}} = 100 \text{ kPa}$$

(41)

For sand number 3 the parameter is: $C_p = 0.43$

- $f_Y$:

$$Y_c = \frac{9 - \sin^2 \varphi_c}{1 - \sin^2 \varphi_c}$$

(7)

$$\bar{Y}^{av} = \frac{Y - 9}{Y_c - 9}$$

$$Y = \frac{27(3 + \eta)}{(3 + 2\eta)(3 - \eta)}$$

$$f_Y = \exp[C_Y \bar{Y}^{av}]$$

(42)

For sand number 3 the parameters are: $\varphi_c = 31.2, C_Y = 2.0$

In this model, the value for $\eta$ is a constant and $\eta^{av} = 0.75$ is used.
• $f_π$:

$$f_π = 1\quad (44)$$

Now the rate of accumulated strain is known, but in order to simulate the results, the accumulation strain is needed and is calculated as follows:

For $N = 1$:

$$\varepsilon^{\text{acc}}(N) = \dot{\varepsilon}^{\text{acc}}(N)\quad (56)$$

For $N > 1$:

$$\varepsilon^{\text{acc}}(N) = \dot{\varepsilon}^{\text{acc}}(N) + \varepsilon^{\text{acc}}(N - 1)\quad (57)$$

(The total accumulation strain is the summation of the accumulation strain rate of each increment from $N = 1$ until the final cycle 1000000 is reached).

The direction of accumulation is clarified:

• $m$:

$$\dot{\varepsilon}^{\text{acc}} = \dot{\varepsilon}^{\text{acc}} m\quad (29)$$

$$m = \frac{\dot{\varepsilon}^{\text{acc}}}{\| \dot{\varepsilon}^{\text{acc}} \|} = (\dot{\varepsilon}^{\text{acc}})^{\rightarrow\rightarrow}\quad (58)$$

This is a unit tensor.

The flow rule of the Modified Cam Clay (MCC) model is used for $m$:

$$m = \left[ \frac{1}{3} \left( p - \frac{q^2}{M^2 p} \right) 1 + \frac{3}{M^2} \sigma \right]^{\rightarrow\rightarrow}\quad (46)$$

$$\begin{bmatrix}
m_{11} \\
m_{22} \\
m_{33} \\
m_{12} \\
m_{13} \\
m_{23}
\end{bmatrix} = \frac{1}{6} \begin{bmatrix}
1 & 1 & \sigma_{11} - p \\
1 & 1 & \sigma_{22} - p \\
1 & 1 & \sigma_{33} - p \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

$$\left[ m_{11}^2 + m_{22}^2 + m_{33}^2 + m_{12}^2 + m_{13}^2 + m_{23}^2 \right]^{\rightarrow\rightarrow}\quad (49)$$

The superposed arrow denotes Euclidean normalization (Annex 1).

For $p$ the value of $p^\text{av}$ is used.
The strain accumulation with the flow rule is as follows:

\[
\dot{\varepsilon}_{11}^{acc} \quad \dot{\varepsilon}_{22}^{acc} \quad \dot{\varepsilon}_{33}^{acc} \quad \dot{\varepsilon}_{12}^{acc} \quad \dot{\varepsilon}_{13}^{acc} \quad \dot{\varepsilon}_{23}^{acc} = \dot{\varepsilon}^{acc} \begin{bmatrix} m_{11} \\ m_{22} \\ m_{33} \\ m_{12} \\ m_{13} \\ m_{23} \end{bmatrix}
\]

(50)

To be able to reproduce all the graphs, the strain accumulation tensor needs to be transformed as a scalar, according to the following formula:

\[\varepsilon = \sqrt{(\varepsilon_{11}^{acc})^2 + 2 (\varepsilon_{33}^{acc})^2}\]

(59)

With \(\varepsilon_1\) and \(\varepsilon_3\) being calculated as followed:

For \(N = 1\):

\[\varepsilon_{11}^{acc}(N) = \dot{\varepsilon}_{11}^{acc}(N)\]

(60)

\[\varepsilon_{33}^{acc}(N) = \dot{\varepsilon}_{33}^{acc}(N)\]

(61)

For \(N > 1\):

\[\varepsilon_{11}^{acc}(N) = \dot{\varepsilon}_{11}^{acc}(N) + \varepsilon_{11}^{acc}(N - 1)\]

(62)

\[\varepsilon_{33}^{acc}(N) = \dot{\varepsilon}_{33}^{acc}(N) + \varepsilon_{33}^{acc}(N - 1)\]

(63)
2.4 Limitations

This model includes a lot of parameters and effects, but it is still a simplified version of the reality, meaning there are some limitations to it.

2.4.1 Miner rule

One limitation is the assumption that the cyclic strain path is regular and smooth and remains constant over a large number of cycles. This can be a problem when:

- The amplitude and direction of loading is changed frequently
- Multiple load sources, causing multiple oscillations, each with varying amplitude or frequency
- A load pattern that consists of simultaneous or sequential events of cyclic and monotonic loading

To tackle these problems, the compliance of the HCA model with the Miner rule was validated. The Miner rule is mostly known from material fatigue calculations [19]. The rule says that the effect of multiple batches of cycles with different stress amplitudes is independent of the sequence of the batches.

The Miner rule has only been validated for uni-axial loading (triaxial test) on freshly pluviated sand samples (no cyclic history). For other, more difficult load cases, the results seem to disobey the Miner rule. In the case of cyclic pre-loading, the Miner rule is also violated [13].

2.4.2 Historiotropy $g^A$

The parameter $g^A$ takes the history of cyclic straining (‘memory’ of the soil) into account. In reality, each load cycle modifies the orientation of the grains and the grain contacts slightly, which increases the resistance of the soil against subsequent load cycles. This process is not fully understood yet, but it is assumed that there is a relationship between the rate of accumulation and the amount and magnitude of historic cycles. Researchers are trying to add a parameter to introduce the changes and reorientations of the grains into the model, since the current scalar that is used is too simplistic to represent the actual process.

Another problem is determining the current state of cyclic strain memory from in-situ or laboratory tests, if the sample is not freshly pluviated (zero cyclic memory). If the soils have been loaded in the past, but $g^A$ is assumed to be zero, the settlements are overestimated when calculated with the HCA model.
3 TESTED MATERIAL

The cyclic tests were performed on quartz sand with subangular grains (figure 23), the constitutive model was based on the properties of sand number 3 (figure 24), a uniform medium coarse to coarse sand (figure 24). The specimens were prepared by pluviating dry sand out of a funnel through air into split moulds (figure 25). The model has a clear set of soil parameters as input, to easily determine the strain accumulation.

Figure 23: Shape of sand grains
[10]

Figure 24: Tested grain size distribution curves
[15]
The main characteristics of the grain size distributions are given in Table 1 and 2 and exist of the following parameters:

- The mean grain diameter $d_{50}$ (figure 26), this is an important particle size distribution parameter, it is the diameter at which 50% of the sample’s mass is comprised of particles that have a diameter which is less. (Also $d_{10}, d_{60}$ and $d_{90}$ are used and have the same way of determination).

- The non-uniformity index ($U$) is based on these particle size distribution parameters

$$U = \frac{d_{60}}{d_{10}}$$  \hspace{1cm} (64)

- The curvature index ($C$) is also based on these particle size distribution parameters

$$C = \frac{d_{60}^2}{d_{60} d_{10}}$$  \hspace{1cm} (65)

The extreme void ratios are also an important parameter and are defined in the German standard code DIN 18126 (1996), this standard was chosen, since it guarantees that no grain-crushing will occur.

- Maximum void ratio ($e_{\text{max}}$) is the void ratio when the soil is packed the loosest, this is done by loosely pouring it into a cylinder (diameter 71 mm, height 112 mm) using a funnel with an outlet diameter of 12 mm.

- Minimum void ratio ($e_{\text{min}}$) is the void ratio when the soil is packed the densest (further compaction is not possible), this value is ‘calculated’ by performing a layer-wise compaction (by applying several lateral hits against the wall of the cylinder) of initially loose saturated sand (Proctor test). At this state, the unit weight has its maximum possible value.
Another important input parameter in the model is the critical friction angle or angle of repose \( (\phi_c) \), it is the mean inclination value of the cone of 10 cone pluviation tests. The angle that is formed (figure 27) is the steepest angle of dip, relative to the horizontal plane to which a material can be piled without 'collapsing'. At this angle, the material on the slope surface is on the verge of sliding.

![Figure 27: Critical friction angle](image)

**Table 1: Characteristics of sand number 3**

<table>
<thead>
<tr>
<th>Soil #</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{50} ) [mm]</td>
<td>0.55</td>
</tr>
<tr>
<td>U</td>
<td>1.8</td>
</tr>
<tr>
<td>C</td>
<td>1.2</td>
</tr>
<tr>
<td>( e_{max} = e_{ref} )</td>
<td>0.874</td>
</tr>
<tr>
<td>( e_{min} )</td>
<td>0.577</td>
</tr>
<tr>
<td>( \phi_c ) [°]</td>
<td>31.2</td>
</tr>
</tbody>
</table>

To validate the model (that was optimised based on the characteristics of sand number 3), it was checked (coded) for the other 7 sands, the main characteristics are listed below (table 2).

**Table 2: Characteristics of sands**

<table>
<thead>
<tr>
<th>Soil #</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{50} ) [mm]</td>
<td>0.15</td>
<td>0.35</td>
<td>0.84</td>
<td>1.45</td>
<td>4.4</td>
<td>0.55</td>
<td>0.52</td>
</tr>
<tr>
<td>U</td>
<td>1.4</td>
<td>1.9</td>
<td>1.4</td>
<td>1.4</td>
<td>1.3</td>
<td>3.2</td>
<td>4.5</td>
</tr>
<tr>
<td>C</td>
<td>0.9</td>
<td>1.2</td>
<td>1.0</td>
<td>0.9</td>
<td>1.1</td>
<td>1.1</td>
<td>0.7</td>
</tr>
<tr>
<td>( e_{max} = e_{ref} )</td>
<td>0.992</td>
<td>0.930</td>
<td>0.878</td>
<td>0.886</td>
<td>0.851</td>
<td>0.811</td>
<td>0.691</td>
</tr>
<tr>
<td>( e_{min} )</td>
<td>0.612</td>
<td>0.544</td>
<td>0.572</td>
<td>0.574</td>
<td>0.622</td>
<td>0.453</td>
<td>0.383</td>
</tr>
<tr>
<td>( \phi_c ) [°]</td>
<td>32.0</td>
<td>32.7</td>
<td>32.9</td>
<td>33.2</td>
<td>37.2</td>
<td>33.1</td>
<td>34.2</td>
</tr>
</tbody>
</table>
The parameters $C_{N1}$, $C_{N2}$ and $C_{N3}$ are calculated based on all three test series that are used to calculate $C_e$, $C_p$ and $C_Y$ (see chapter 2.2.2) and sometimes with an additional test series with different stress amplitudes, to choose an appropriate amplitude-pressure ratio for the other three test series.

The granulometric soil parameters are not all directly used as an input parameter (only the critical friction angle and the minimum and maximum void ratios). The process of determining the input parameters is quite laborious and sophisticated test devices are needed multiple times. To simplify this procedure, some correlations between the material parameters and the granulometric properties have been carried out (figure 28a, b, c and d).

\[ C_{N1} = 0.0002 \exp(-0.65 \, d_{50}) \exp(0.91 \, U) \]  
\[ C_{N2} = 0.95 \exp(0.33 \, d_{50}) \exp(-0.90 \, U) \]  
\[ C_{N3} = 0.00003 \exp(-0.69 \, d_{50}) \exp(0.26 \, U) \]  
\[ C_e = 0.96 \, e_{\text{min}} \]

The parameters $C_Y$ and $C_P$ are independent of the granulometric properties, so the recommendations are to use constants values, the way they are calculated is already explained in chapter 2.2.2. [16]

For $N = 10^5$ the parameters are given in Table 3 and 4.
Table 3: Parameters of the HCA model

\[11, 15\]

<table>
<thead>
<tr>
<th>Sand</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{N1})</td>
<td>0.00036</td>
</tr>
<tr>
<td>(C_{N2})</td>
<td>0.43</td>
</tr>
<tr>
<td>(C_{N3})</td>
<td>0.00005</td>
</tr>
<tr>
<td>(C_p)</td>
<td>0.43</td>
</tr>
<tr>
<td>(C_Y)</td>
<td>2.0</td>
</tr>
<tr>
<td>(C_e)</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Table 4: Parameters of the HCA model

\[15\]

<table>
<thead>
<tr>
<th>Sand</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{N1})</td>
<td>0.00087</td>
<td>0.00077</td>
<td>0.00027</td>
<td>0.00043</td>
<td>0.000048</td>
<td>0.0044</td>
<td>0.0083</td>
</tr>
<tr>
<td>(C_{N2})</td>
<td>0.22</td>
<td>0.27</td>
<td>0.36</td>
<td>0.32</td>
<td>1.27</td>
<td>0.29</td>
<td>0.059</td>
</tr>
<tr>
<td>(C_{N3})</td>
<td>0.00004</td>
<td>0.0000076</td>
<td>0.000004</td>
<td>0.0000007</td>
<td>0.0</td>
<td>0.00005</td>
<td>0.00007</td>
</tr>
<tr>
<td>(C_p)</td>
<td>0.60</td>
<td>0.84</td>
<td>0.58</td>
<td>0.68</td>
<td>0.30</td>
<td>0.68</td>
<td>0.61</td>
</tr>
<tr>
<td>(C_Y)</td>
<td>1.8</td>
<td>2.7</td>
<td>2.8</td>
<td>2.8</td>
<td>3.0</td>
<td>2.2</td>
<td>3.1</td>
</tr>
<tr>
<td>(C_e)</td>
<td>0.57</td>
<td>0.55</td>
<td>0.56</td>
<td>0.54</td>
<td>0.38</td>
<td>0.44</td>
<td>0.36</td>
</tr>
</tbody>
</table>

The values for \(A\) in equation 26 are replaced by \(A^{av}\) (curve-fitting, using equation 52) and are listed in table 5.

Table 5: Average A-value

<table>
<thead>
<tr>
<th>Sand</th>
<th>A (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1031.98831</td>
</tr>
<tr>
<td>2</td>
<td>635.2410782</td>
</tr>
<tr>
<td>3</td>
<td>919.8245965</td>
</tr>
<tr>
<td>4</td>
<td>1056.717093</td>
</tr>
<tr>
<td>5</td>
<td>957.8140536</td>
</tr>
<tr>
<td>6</td>
<td>1119.657258</td>
</tr>
<tr>
<td>7</td>
<td>1076.10175</td>
</tr>
<tr>
<td>8</td>
<td>1370.139282</td>
</tr>
</tbody>
</table>

Table 6: A-value

\[q\text{amp}^{l}\] | A         |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>865.1242669</td>
</tr>
<tr>
<td>22</td>
<td>829.2817912</td>
</tr>
<tr>
<td>31</td>
<td>901.5250705</td>
</tr>
<tr>
<td>42</td>
<td>897.0502631</td>
</tr>
<tr>
<td>51</td>
<td>891.2318927</td>
</tr>
<tr>
<td>60</td>
<td>969.8142501</td>
</tr>
<tr>
<td>70</td>
<td>981.1663945</td>
</tr>
<tr>
<td>80</td>
<td>1008.713008</td>
</tr>
</tbody>
</table>

To check whether the results are ‘better’, the results of using \(A^{av}\) are compared with the results when using the \(q\text{amp}^{l}\)-dependent A-value, this is done for sand 3 (table 6).

This is the part of the model that was the most unclear and needs some further investigation in the future.
Another unclear part of the model, are the initial relative densities (Equation 20) that were used in the literature. The initial relative density ($I_{DI}$) is a single value that is used to calculate the void ratio (equation 55), but different ranges were mentioned in the literature, an overview is given in table 7. Since a single value is needed, the values from column 2 (in bold) were used.

Table 7: Overview of all used initial relative density values

<table>
<thead>
<tr>
<th>Sand</th>
<th>$I_{DI}$</th>
<th>$I_{DI}$</th>
<th>$I_{DI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.53</td>
<td>0.51-0.54</td>
<td>0.50-0.54</td>
</tr>
<tr>
<td>2</td>
<td>0.51</td>
<td>0.48-0.53</td>
<td>0.45-0.51</td>
</tr>
<tr>
<td>3</td>
<td>0.60</td>
<td>0.58-0.61</td>
<td>0.57-0.69</td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td>0.64-0.66</td>
<td>0.62-0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.59</td>
<td>0.56-0.61</td>
<td>0.50-0.61</td>
</tr>
<tr>
<td>6</td>
<td>0.77</td>
<td>0.76-0.77</td>
<td>0.62-0.78</td>
</tr>
<tr>
<td>7</td>
<td>0.61</td>
<td>0.56-0.63</td>
<td>0.59-0.63</td>
</tr>
<tr>
<td>8</td>
<td>0.60</td>
<td>0.56-0.63</td>
<td>0.55-0.62</td>
</tr>
</tbody>
</table>
4 SIMULATION OF RESULTS

In order to be able to say that the model is implemented correctly, the graphs that were generated in the papers need to be reproduced. This verification is focused on sand number 3 (figure 24), but in order to evaluate the model more accurately, it is done for all 8 types of sands used in the literature. What is important to know is that the 8 sands used in ([10]) are different from the 8 sands used in ([19]), except sand number 3 is the same in both documents. In this chapter, the 8 sands from [11] are used as input.

4.1 Strain amplitude

The strain amplitude was assumed constant during the 100,000 cycles. The paper was not straightforward to reproduce, since graphs from coded results and from laboratory test results were shown next to each other, without clearly explaining the source of each graph. According to the laboratory test results, the strain amplitude is not constant during the (explicit) cycles (figure 29).

Since the reproduced code only focuses on the explicit mode of calculation, the strain amplitude is assumed constant, it is not evaluated in an implicit control cycle. However, it is reasonable that the strain amplitude reduces with every cycle increment. To check whether the strain amplitude that is calculated in my code is ‘correct’, we compare it with the value of the average strain amplitude in figure 29. The results are similar, yet not identical (see table 8).

<table>
<thead>
<tr>
<th>( q^{ampi} )</th>
<th>Matlab ( [\times 10^{-4}] )</th>
<th>Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.54757</td>
<td>0.5203285</td>
</tr>
<tr>
<td>22</td>
<td>1.0039</td>
<td>1.039796</td>
</tr>
<tr>
<td>31</td>
<td>1.4146</td>
<td>1.462029</td>
</tr>
<tr>
<td>42</td>
<td>1.9163</td>
<td>2.064413</td>
</tr>
<tr>
<td>51</td>
<td>2.3272</td>
<td>2.483179</td>
</tr>
<tr>
<td>60</td>
<td>2.7379</td>
<td>2.903505</td>
</tr>
<tr>
<td>70</td>
<td>3.1942</td>
<td>3.444752</td>
</tr>
<tr>
<td>80</td>
<td>3.6505</td>
<td>4.103121</td>
</tr>
</tbody>
</table>
4.2 Functions

All the functions (equations 34 – 42) are reproduced and individually verified if they are coded the same way as in the literature. This is done with the parameters of sand number 3 (figure 24).

4.2.1 $f_{ampl}$

![Graph of f_ampl](image)

Since the strain amplitude is a constant value (for each sand sample) in the code, $f_{ampl}$ is also a constant value (different values, depending on the situation). Figure 30 cannot be reproduced, since the strain amplitude is not varying in the code. What can be checked is the value of $f_{ampl}$ for these constant strain amplitude values. This is checked for different values of $q_{ampl}$ for sand number 3 (figure 24), the results are listed in table 9.

<table>
<thead>
<tr>
<th>$q_{ampl}$</th>
<th>$e_{ampl} \times 10^{-4}$</th>
<th>$f_{ampl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.54757</td>
<td>0.2998</td>
</tr>
<tr>
<td>22</td>
<td>1.0039</td>
<td>1.0078</td>
</tr>
<tr>
<td>31</td>
<td>1.4146</td>
<td>2.0010</td>
</tr>
<tr>
<td>42</td>
<td>1.9163</td>
<td>3.6730</td>
</tr>
<tr>
<td>51</td>
<td>2.3272</td>
<td>5.4158</td>
</tr>
<tr>
<td>60</td>
<td>2.7379</td>
<td>7.4959</td>
</tr>
<tr>
<td>70</td>
<td>3.1942</td>
<td>10.2027</td>
</tr>
<tr>
<td>80</td>
<td>3.6505</td>
<td>13.3260</td>
</tr>
</tbody>
</table>

At first sight, these values are according to the curve in figure 30 and thus it is concluded that this function is reproduced correctly.
4.2.2 $f_N$

As seen in figure 31a and 31b (identical curves), this part of the model (cyclic preloading) is reproduced correctly.

4.2.3 $f_N$

As seen in figure 32a and 32b (identical curves), this part of the model (cyclic preloading) is reproduced correctly.
4.2.4 \( f_p \)

Figure 33: a) \( f_p \) – paper, b) \( f_Y \) – paper

For sand number 3, the curve of \( f_p \) is recreated, by checking the value of the function at different values of the average mean pressure.

The values that are produced (table 10) seem to be the same as the corresponding values at the curve (figure 33a).

4.2.5 \( f_Y \)

This function’s only varying factor is ‘\( \eta \)’ (equation 5 and 9), this curve is recreated, by checking the value of the function at different values of the average stress ratio (table 11).

<table>
<thead>
<tr>
<th>( p^{\text{av}} )</th>
<th>( f_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5373</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>200</td>
<td>0.6505</td>
</tr>
<tr>
<td>300</td>
<td>0.4232</td>
</tr>
<tr>
<td>400</td>
<td>0.2753</td>
</tr>
</tbody>
</table>

Table 10: \( f_p \) – Matlab

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>( \bar{Y}^{\text{av}} )</th>
<th>( f_Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1534</td>
<td>1.3590</td>
</tr>
<tr>
<td>0.75</td>
<td>0.3408</td>
<td>1.9771</td>
</tr>
<tr>
<td>1</td>
<td>0.6135</td>
<td>3.4106</td>
</tr>
<tr>
<td>1.125</td>
<td>0.7887</td>
<td>4.8426</td>
</tr>
<tr>
<td>1.25</td>
<td>0.9959</td>
<td>7.3282</td>
</tr>
</tbody>
</table>

Table 11: \( f_Y \) – paper

The values that are produced (table 11) seem to be exactly the same as the corresponding values at the curve (figure 33b).
4.2.6 $f_e$

This function’s only varying factor is $e$ (equation 40), this curve is recreated, by checking the value of the function at different values of the void ratio (table 12).

![Figure 34: $f_e$ – paper]

<table>
<thead>
<tr>
<th>$l_{DI}$</th>
<th>$e$</th>
<th>$f_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.577</td>
<td>0.0146</td>
</tr>
<tr>
<td>0.75</td>
<td>0.6513</td>
<td>0.1259</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7255</td>
<td>0.3350</td>
</tr>
<tr>
<td>0.25</td>
<td>0.7998</td>
<td>0.6298</td>
</tr>
<tr>
<td>0</td>
<td>0.874</td>
<td>1</td>
</tr>
</tbody>
</table>

The values that are produced (table 12) seem to be exactly the same as the corresponding values at the curve (figure 34). For $e_{\text{max}}$, the exact same value is achieved as in figure 34.

4.2.7 $f_\pi$

This ‘function’ is a constant value in the case of uniaxial 1D cycles ($f_\pi = 1$).
4.3 Strain accumulation

The strain accumulation (calculated with equation 59) is the main component of this model, so the results are important. In this chapter, for each sand and for every used value of \( q^{ampl} \) are the results from the code compared to the results from the literature.

**Sand 1**

![Figure 35: Accumulated strain - Sand 1 – Paper [15]](image)

**Figure 36: a) Accumulated strain - Sand 1 - Qampl 19 - Matlab, b) Accumulated strain - Sand 1 - Qampl 40 - Matlab, c) Accumulated strain - Sand 1 - Qampl 59 - Matlab**
For the three different load cases of sand number 1, three different curves are obtained, in comparison to the corresponding graphs from the literature. For lower values of $q_{\text{ampl}}$, the code is underestimating the accumulated strain and for higher values of $q_{\text{ampl}}$ it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100.000$ is changing, for the lowest $q_{\text{ampl}}$ it is clearly not the same, for a higher value it is almost the same, but for the highest value it is a lot different. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.

**Sand 2**

![Figure 37: Accumulated strain - Sand 2 – Paper](image)

![Figure 38: a) Accumulated strain - Sand 2 - Qampl 15 – Matlab, b) Accumulated strain - Sand 2 - Qampl 28 – Matlab](image)

![Figure 39: a) Accumulated strain - Sand 2 - Qampl 36 – Matlab, b) Accumulated strain - Sand 2 - Qampl 46 – Matlab](image)
For the eight different load cases of sand number 2, two different curves are obtained, in comparison to the corresponding graphs from the literature.

For lower values of $q_{\text{ampl}}$ (15 – 46), the code is underestimating the accumulated strain and for higher values of $q_{\text{ampl}}$ (57 – 87) it is overestimating the accumulated strain.

The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q_{\text{ampl}} = 57$, there is a change in which curve is generating the highest value.

The shape (trend) of the curves is similar in all cases, but it is clearly not identical.
Sand 3

Figure 42: Accumulated strain - Sand 3 – Paper

[15]

Figure 43: a) Accumulated strain - Sand 3 - Qampl 12 – Matlab, b) Accumulated strain - Sand 3 - Qampl 22 – Matlab

Figure 44: a) Accumulated strain - Sand 3 - Qampl 31 – Matlab, b) Accumulated strain - Sand 3 - Qampl 42 - Matlab
For the eight different load cases of sand number 3, two different curves are obtained, in comparison to the corresponding graphs from the literature.

For lower values of $q_{ampl}$ (12 – 51), the code is underestimating the accumulated strain and for higher values of $q_{ampl}$ (60 – 80) it is overestimating the accumulated strain.

The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q_{ampl} = 60$, there is a change in which curve is generating the highest value.

The shape (trend) of the curves is similar in all cases, but it is clearly not identical.
For sand number 3, the influence of using a non-constant value for $A$ instead of $A^{av}$ is checked, using the values from table 5.

Figure 47: a) Accumulated strain - Sand 3 - Qampl 12, b) Accumulated strain - Sand 3 - Qampl 22

Figure 48: a) Accumulated strain - Sand 3 - Qampl 31, b) Accumulated strain - Sand 3 - Qampl 42

Figure 49: a) Accumulated strain - Sand 3 - Qampl 51, b) Accumulated strain - Sand 3 - Qampl 60
For the eight different load cases of sand number 3, the curves all have the same shape. The difference between the code and the literature for the accumulated strain at \( N = 100,000 \) is almost zero, for each load case. The shape (trend) of the curves is similar in all cases, but it is clearly not identical. It is however obviously visible that this approach results in better curves.

Sand 4

Figure 50: a) Accumulated strain - Sand 3 - Qampl 70, b) Accumulated strain - Sand 3 - Qampl 80

Figure 51: Accumulated strain - Sand 4 – Paper

Figure 52: a) Accumulated strain - Sand 4 - Qampl 20 – Matlab, b) Accumulated strain - Sand 4 - Qampl 40 – Matlab
For the four different load cases of sand number 4, two different curves are obtained, in comparison to the corresponding graphs from the literature. For lower values of $q_{ampl}$ (20, 40), the code is underestimating the accumulated strain and for higher values of $q_{ampl}$ (60, 80) it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q_{ampl} = 60$, there is a change in which curve is generating the highest value. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.

**Sand 5**

![Figure 54: Accumulated strain - Sand 5 – Paper](image)
Numerical studies of a high-cycle accumulation model for sand

Figure 55: a) Accumulated strain - Sand 5 - Qampl 10 – Matlab, b) Accumulated strain - Sand 5 - Qampl 20 – Matlab

Figure 56: a) Accumulated strain - Sand 5 - Qampl 30 – Matlab, b) Accumulated strain - Sand 5 - Qampl 40 – Matlab

Figure 57: a) Accumulated strain - Sand 5 - Qampl 50 – Matlab, b) Accumulated strain - Sand 5 - Qampl 60 – Matlab
For the eight different load cases of sand number 5, the curves resulting from the code are not really the same as the graphs from the literature. For lower values of $q^{ampl}$ (10, – 30), the code is underestimating the accumulated strain and for higher values of $q^{ampl}$ (40 – 80) it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q^{ampl} = 40$, there is a change in which curve is generating the highest value. The shape (trend) of the curves is not similar to the literature in all cases.

Sand 6
For the three different load cases of sand number 6, similar curves are obtained, in comparison to the corresponding graphs from the literature. For lower values of $q_{\text{amp}}$, the code is underestimating the accumulated strain and for higher values of $q_{\text{amp}}$ it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q_{\text{amp}} = 60$, there is a change in which curve is generating the highest value. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.

Sand 7
For the four different load cases of sand number 7, two different curves are obtained, in comparison to the corresponding graphs from the literature.

For lower values of $q_{\text{ampl}}$ (20 – 59), the code is underestimating the accumulated strain and for higher values of $q_{\text{ampl}}$ (78) it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100.000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q_{\text{ampl}} = 78$, there is a change in which curve is generating the highest value. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.
To validate the model more completely, other graphs are also modelled with an adapted version of the code, but using the same parameters. If these curves do not compare with each other, this means that the code is not correct.

Figure 64: Accumulated strain - Sand 7 - Changing average stress ratio

Figure 65: a) Accumulated strain - Sand 7 - $\eta^{av} 0.5$ – Matlab, b) Accumulated strain - Sand 7 - $\eta^{av} 0.75$ – Matlab

Figure 66: a) Accumulated strain - Sand 8 - $\eta^{av} 1.0$ – Matlab, b) Accumulated strain - Sand 7 - $\eta^{av} 1.125$ – Matlab, c) Accumulated strain - Sand 8 - $\eta^{av} 1.25$ – Matlab
For the five different load cases of sand number 7, two different curves are obtained, in comparison to the corresponding graphs from the literature.

For lower values of $q^{\text{ampl}}$ (20 – 59), the code is underestimating the accumulated strain and for higher values of $q^{\text{ampl}}$ (78) it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q^{\text{ampl}} = 78$, there is a change in which curve is generating the highest value. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.

To check if the use of $A^{av}$ is correct, the results are compared to when $A = 467$.

---

**Figure 67:** a) Accumulated strain - Sand 7 - $\eta^{av} 0.5$ – $A = 467$, b) Accumulated strain - Sand 7 - $\eta^{av} 0.75$ – $A = 467$

**Figure 68:** a) Accumulated strain - Sand 7 - $\eta^{av} 1.0$ – $A = 467$, b) Accumulated strain - Sand 7 - $\eta^{av} 1.125$ – $A = 467$, c) Accumulated strain - Sand 7 - $\eta^{av} 1.25$ – $A = 467
For the five different load cases of sand number 7, two different curves are obtained, in comparison to the corresponding graphs from the literature. For lower values of $q^{\text{amp}}$ (20 – 59), the code is underestimating the accumulated strain and for higher values of $q^{\text{amp}}$ (78) it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q^{\text{amp}} = 78$, there is a change in which curve is generating the highest value. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.

Sand 8

![Figure 69: Accumulated strain - Sand 8 – Paper

Figure 70: a) Accumulated strain - Sand 8 - Qampl 13 – Matlab, b) Accumulated strain - Sand 8 - Qampl 21 – Matlab

Figure 71: a) Accumulated strain - Sand 8 - Qampl 31 – Matlab, b) Accumulated strain - Sand 8 - Qampl 41 – Matlab]
For the eight different load cases of sand number 8, similar curves are obtained, in comparison to the corresponding graphs from the literature. For lower values of $q_{ampl}$ (13 – 50), the code is underestimating the accumulated strain and for higher values of $q_{ampl}$ (59 – 78) it is overestimating the accumulated strain. The difference between the code and the literature for the accumulated strain at $N = 100,000$ is changing consistently, the higher the load amplitude, the higher the difference. Although at $q_{ampl} = 78$, there is a change in which curve is generating the highest value. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.
To validate the model more completely, other graphs are also modelled with an adapted version of the code, but using the same parameters. If these curves do not compare with each other, this means that the code is not correct.

Because the graphs are overlapping, only the data from $p_{av} = 200, 250$ and $300 \text{ kPa}$ is converted to Matlab, using the DigXY software.

The difference between the code and the literature for the accumulated strain at $N = 100,000$ is almost constant for every case. The shape (trend) of the curves is similar in all cases, but it is clearly not identical.
To check if the use of $A^{av}$ is correct, the results are compared to when $A=467$.

Figure 76: a) Accumulated strain - Sand 8 - Pav 250 - $A=467$, b) Accumulated strain - Sand 8 - Pav 200 - $A=467$, c) Accumulated strain - Sand 8 - Pav 300 - $A=467$

The difference between the code and the literature for the accumulated strain at $N = 100.000$ is almost constant for every case. The shape (trend) of the curves is similar in all cases, but it is clearly not identical. The results are clearly worse, than if $A^{av}$ is used.
4.4 Discussion of results

In this work, there are some assumptions made in order to be able to get the results that were presented in the previous chapters. These assumptions together with parameters that were missing in the literature will be explained in this chapter and afterwards the results will be discussed.

Assumptions:

- The strain amplitude stays constant during the increasing cycles (conform the explicit method), but in reality also the implicit method should be used, to recalculate the strain amplitude (control-cycles), after the first cycle is neglected (figure 4).

- Another ‘problem’ that was encountered is the fact that the strain amplitude is constant during the explicit calculations, while figure 26 shows a curve where the strain amplitude decreases with increasing number of cycles. This graph however is based on the results of soil tests, not from the model, so this is a misleading graph.

- The void ratio $e$ stays constant during the cycles (meaning $f_e$ also stays constant), in reality the void ratio will change each cycle, unless an extrema is reached ($e_{min}$ or $e_{max}$).

- The formula to calculate the elastic stiffness and the bulk modulus are different in the literature, one paper uses ‘$p$’ while another one uses ‘$p_{av}$’. This could also lead to different results. In this code, $p_{av}$ is used in all formulas where $p = p_{av} + p_{ampl}$ was an input parameter.

- The values of $p$ and $q$ are constant, meaning $f_p$ and $f_Y$ stay constant during the cyclic loading.

Parameters missing:

- The exact used value of initial relative density is unclear, there were multiple values mentioned in the literature (table 6) and sometimes a range is given instead of a constant value, the values that were used in the code are put in bold. This is also a constant value, because the void ratio is not changing in this code. If the void ratio changes, the relative density will change as well.

- Poisson’s ratio is estimated at a value of 0.2, this is a value that is realistic for this situation, but it is not confirmed in the literature that the model is in fact using the same value.

- In the literature there is some information about the stiffness that is used, but in the paper that is being used as a basis for the code [15], nothing is mentioned about the stiffness. The biggest assumption that is made for the stiffness is the value of ‘$A$’ used in equation 26. It is mentioned in literature that further research regarding the stiffness has to be executed. Also the stiffness is stress-dependent, while it is constant in this code. The assumption of a constant $A_{av}$ for each
sand sample instead of ‘A’ constant for the model, is important however, because without this assumption, the equations seemed to be reproduced correctly, but the results were not according to the literature (figure 84a versus figure 84b).

With all these assumptions being made, it is not possible to expect the exact results as in the literature. In general the shape of the reproduced curves is almost the same as the graphs from the paper.

Now that it is known that there will not be a perfect fit between the literature and the code, the results are discussed in detail.

**Strain amplitude:**

- The results are similar, yet not identical (see table 8). There is almost a consistent difference, only for $q_{\text{amp}} = 12 \text{ kPa}$ there is a slightly larger value than the average from the literature, in all other cases, there is a slightly smaller value.

- Another trend that is significant is that the higher the $q_{\text{amp}}$, the higher the difference is between the literature and de code. This could imply that there is another influence on the strain amplitude that is neglected in the code.

**Functions:**

- $f_{\text{amp}}$

  At first sight, these values (table 9) are according to the curve in figure 30 and thus it is concluded that this function is reproduced correctly. However, it is not possible to accurately compare the values with the curve, since the margin is so big. And since the strain amplitude is so small, a small mistake will lead to a small mistake in the function (without having a big influence on the end result, being the strain accumulation).

- $\dot{j}_N$

  As seen in figure 31a and 31b, this part of the model (cyclic preloading) is reproduced correctly, the curve is identical, meaning that the influence of the historiotropy is reproduced correctly.
• $f_N$

As seen in figure 31a and 31b, this part of the model (cyclic preloading) is reproduced correctly, the curve is identical, meaning that the influence of the historiotropy is reproduced correctly.

• $f_p$

The values that are produced (table 10) seem to be the same as the corresponding values at the curve (figure 33a). There isn’t a noticeable difference, so it is concluded that this function is reproduced correctly.

• $f_r$

The values that are produced (table 11) seem to be exactly the same as the corresponding values at the curve (figure 33b). Since the code isn’t directly based on $\bar{Y}^{av}$, but on $\eta$, it isn’t possible to directly compare the curve with the average stress ratio $\bar{Y}^{av}$. But there isn’t a noticeable difference with the values available, so it is concluded that this function is reproduced correctly.

• $f_e$

The values that are produced (table 12) seem to be exactly the same as the corresponding values at the curve (figure 34). For $e_{max}$, the exact same value is achieved as in figure 34. However, since the void ratio should be changing during each cycle, the function should also change each cycle, this effect cannot be checked by comparing the results. Therefore another method is needed.

• $f_\pi$

This ‘function’ is a constant value in the case of uniaxial 1D cycles ($f_\pi = 1$).

• $m$

The functions of the model are all reproduced correctly, according to chapter 4.2.1 - 4.2.7. The only influence that cannot be checked with the literature is that from the direction of accumulation ‘$m$’, so the influence is searched by comparing the ‘scalar’ (equation 31 without multiplication by ‘$m$’) with the scalar made from the tensor (including the direction of accumulation), using equation 59. As seen in figure 77a and 77b, there is no difference.
Strain accumulation:

- The differences between the code and the literature is bigger for higher stress amplitudes. For lower stress amplitude the strain accumulation is less than the paper (figure 36a), while for larger stress amplitudes the strain accumulation is more than the paper (figure 39b). This trend is in according with the variation in $A$. This phenomenon is seen for all sands, the value at which the graph from the code is overestimating does change however, but is mostly around 60 kPa. Especially the difference at $N = 100.000$ is significant.

- If there are a ‘low’ number of cycles, the curves from the code are almost generating the same values as the literature, but at larger values of ‘$N$’ the code is generating higher values (figure 44b).

- Sand number 5 has different graphic trends then the rest of the soil samples that were used. The shape is different, resulting in an underestimation when $N$ is ‘small’ and an overestimation when $N$ is ‘large’ (figure 55a). It can be concluded that this sand’s reproduction is the worst of all the 8 sands, however the reason why is not straightforward.

- It is clear to see that with the use of $A^{av}$, the results are better, yet not perfectly similar to the literature (figure 84 versus 85). The problem that occurs when using $A^{av}$, is that it isn’t constant for each sand, meaning that there should be another influencing parameter that needs changes, in order to simulate the same results as the literature. To check, the difference between using $A^{av}$ and $A$ (also changed by the scale factor, but changing with $q^{ampi}$, table 6) is evaluated. It is clearly visible that if a non-constant (for the sample) value is used (according to $q^{ampi}$), that better results are achieved than if the value is constant for the sample. The curves are still not perfectly the same, but the values at $N = 100.000$ are very similar, only the trend of the curves differ a bit (figures 43a – 46b versus figures 47a - 50b).

- For sand 7 and sand 8, other influences then $q^{ampi}$ are checked ($\eta^{av}$ for sand 7 and $p^{av}$ for sand 8). In both graphs, similar results are obtained, meaning that the code is also capable of reproducing their influence.
Reasons why the fit becomes worst in certain cases:

- The initial density and the void ratio are the main ‘problem’, since they are varying for every sand, but it is not known what the relation is regarding the number of cycles. That’s why a mean value was used in the code. What we would expect is a decrease in void ratio with increasing stress amplitude. The results we have now confirm this hypothesis: with lower stress amplitude the strain accumulation predicts lower values than expected and with increasing cycles, the accumulation is higher.

- For higher stress amplitude the difference in strain accumulation between the Matlab code and the literature increases.

The general difference in curves can be caused by multiple reasons:

- The assumptions that are being made.

- The parameters that are unknown and thus ‘estimated’.

- The way the results from the literature are being implemented in Matlab, there are some errors using the DigXY software, the curves are not as smooth as in reality and the values may also differ (small error).

- A mistake in the Matlab code that is looked over.

- A mistake in the written version of the paper, that is not in the code of the paper (unlikely).
5 CONCLUSION AND OUTLOOK

5.1 Conclusion

HCA model:
In this thesis, the HCA model by Niemunis A. and Wichtmann T. was explained and reproduced in Matlab software. The constitutive equation is used to perform accurate explicit calculations of the accumulation of residual deformations when sand is subjected to cyclic loading with a high number of cycles \( N > 10^5 \) and a relatively low strain amplitude \( \varepsilon^{ampl} < 10^{-3} \). The empirical equations are valid, assuming that the rate of accumulation is only dependent of a number of material properties. The direction of accumulation is solely dependent on the stress state \( \eta \), the intensity of accumulation is dependent on the strain amplitude \( \varepsilon^{ampl} \), number of cycles \( N \), cyclic preloading \( g^A \), void ratio \( e \), average mean pressure \( p^{av} \) and average stress ratio \( \eta^{av} \). These assumptions were validated (for limited cases) based on the results of numerous cyclic triaxial tests.

Results:
The HCA model was not as easy to implement as first thought, since the available literature did not give all the unknown parameters in a straightforward manner. The model was reproduced as a Matlab code and the results were compared with the results from the literature. The shape of the curves were similar but not identical, this phenomenon can be due to different reasons, which are explained in detail in chapter 4.4. The Matlab code is best able to reproduce the results from sand number 3, this is as expected, since the model is based around the test results from sand number 3 (figure 24).

Personal:
Looking back at the main objective of this thesis: ‘Understanding the High Cycle Accumulation model and performing a numerical study by replicating the output data from the literature, using Matlab software.’ The following conclusions can be drawn: the model is understood relatively well and future research regarding this topic can be performed. The constitutive equations were, despite the assumptions and missing data successfully reproduced in Matlab. The reason why the results are not perfect are understood and explained, with more information about them, the model can be optimised.

Looking back at the past year, not all the initially planned goals were reached. The goal of the thesis was changed during the semester, due to a lack of time and due to unforeseen difficulties in reproducing the model. Although the results are not as good as hoped, I am pleased with what I have achieved. I have learned a lot as an exchange student at another university and I have been through an evolution as a ‘researcher’. In the beginning it was hard to try and understand the HCA model and to execute the things that were expected from me, but at the end I was able to critically evaluate the comments from my supervisors and even propose actions and evaluate intermediate results myself. I am glad that I chose a subject in the field of geotechnical engineering, since Aalto University is known for this department and I am happy to have been part of it (for a short time).
5.2 Outlook

Since the initial end goal was changed during the semester and the scope of the subject is very wide, a lot of things can be added to this thesis/code, this chapter gives a short overview of the different possibilities.

5.2.1 Model

The present literature can answer several questions concerning the material behaviour of non-cohesive soils under cyclic loading, further research is however needed for some aspects to become clear.

The model I reproduced in Matlab can be extended with the available research on the High-Cycle accumulation model, this extension can include:

- Membrane penetration (membrane around the sample is being pressed into the voids between the grains due to a variation of the effective lateral stress).
- Out of phase cycles (components oscillate with a phase-shift in time)
- Polarization changes ($f_\pi \neq 1$)
- Changing the code to simulate undrained cyclic tests (focus on liquefaction)
- Focusing on the implicit method (using FEM, see section 5.2.2)
- Changing the code to produce stress accumulation instead of strain accumulation
- Multiaxial stress cycles
- Higher number of cycles

Besides the available literature, some future research suggestions can be proposed:

- Checking the effect of larger numbers of cycles, smaller/higher average stresses, larger strain amplitudes, checking the model for different sands, using different test methods, providing correlations with more material constants ($C, U, ...$).
- It is also very important to have a correct value of the elastic stiffness $E$, since it connects the accumulated stress with the accumulated strain. In this work the value of the elastic stiffness is not calculated, but estimated in each case, based on the results from the literature, this needs to be studied more. Also the strain amplitude is not calculated in the implicit cycles but in the explicit ones.
- An efficient in-situ method to determine the historirotropy has to be developed.
- If the model is ‘completed’ for non-cohesive soils, explicit relations for the cyclic behaviour of cohesive soils can be investigated.

In the model I implemented, the plastic part of strain was omitted, but if more realistic results are wanted, this should be used. But this step should be straightforward, using a finite element method (FEM) program. In a next phase, the finished model can be used to calculate, for example the settlements of off-shore foundations of wind turbines. If this model is verified for more samples and more test methods,
this could become a standard way of calculating in the industry in the future, but for now some more research and results are needed.

### 5.2.2 Finite element method

Because of the lack of time this model was not implemented in the finite element program ‘Thebes’, containing elastic and elastoplastic constitutive models, made by Dr Ayman Abed. But of course it is possible to use different Finite element programs to do the needed calculations.

In the future this code could be implemented in FEM using figure 78 and the following explicit scheme (Lukkezen T., 2016)

**Implicit phase:**
1. Calculation of the initial stress field (with self-weight and static loads) using a conventional plasticity model.
2. Implicit calculation of the first two load cycles, using a plasticity model where small strain non-linearity of soil is accounted for, the yield surface should not be encountered.
3. Recording of the logarithmic strain path during the first regular cycle at each integration point
4. Evaluation of the tensorial strain amplitude $\mathbf{A}_\varepsilon$ from the recorded strain path in each integration point. This tensor is assumed constant in all of the following load cycles, until recalculation in a control-cycle.

**Pseudo-creep phase:**
1. Calculation of the accumulation rate $\dot{\varepsilon}^{acc}$ in every integration point according to equation 31. The accumulation rate is calculated explicitly and is a function of the strain amplitude, the current load cycle $N$ and a series of soil and stress state parameters. Some of the parameters are updated in every increment and some have the initial conditions throughout the steps.
2. Calculation of the stress rate $\dot{\sigma}$ according to equation 22 and the stress increment $\Delta \sigma = \dot{\sigma} \Delta N$ caused by a loading block of cycles. The plastic strain rate is calculated according to the Matsuoka and Nakai yield condition.
3. The equilibrium conditioned is checked, meaning that the internal and external stresses are compared. If no equilibrium is reached, an iteration step is made by adjusting the stiffness matrix $\mathbf{E}$ and recalculating $\dot{\sigma}$ in the previous step.
4. Repetition of step 1-4 until: a new load block with different strain amplitude is reached (go back to step 2 of implicit phase); a control cycle is reached (continue); the end of the test is reached (stop).

**Control phase:**
5. In the control phase, the strain amplitude is recalculated according to steps 2-4 of the implicit phase. Afterwards the pseudo-creep phase continues.
5.2.3 Cyclic direct simple shear test

In the research of Prof Dr Adam Bezuijen at the University of Ghent and at Deltares, this model can be used to calculate the apparatus stiffness of undrained cyclic direct simple shear laboratory tests (CDSS) (figure 79). Therefore, the model needs to be adapted from drained to undrained and from cyclic triaxial test to cyclic direct simple shear test (focus on horizontal loading). Only if these changes are made will the model be able to calculate the apparatus stiffness. Because of a lack of time this wasn’t executed during this year, but since I will be studying an additional master in Civil Engineering at Ghent University, there is an agreement with Bezuijen A. that I will focus on this during my master thesis for that study program (academic year 2020-2021).

In this procedure there is also a FEM implementation (5.2.2) possible.

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Figure 79: Example of an undrained cyclic simple shear test apparatus [2]
BIBLIOGRAPHY

ANNEXES

Annex 1: Tensors

The tensorial notation is used, for scalar variables characters with normal letters (for example e, N) are used, while second-order tensors are denoted by bold letters (for example \( \sigma, \varepsilon \)). The notation of tensor products is given in table 13.

Table 13: Tensor operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Tensorial notation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dyadic product</td>
<td>( A \otimes B )</td>
<td>Fourth-order tensor</td>
</tr>
<tr>
<td>Single contraction</td>
<td>( A : B )</td>
<td>Second-order tensor</td>
</tr>
<tr>
<td>Double contraction</td>
<td>( A \cdot B )</td>
<td>Scalar</td>
</tr>
<tr>
<td>Quadruple contraction</td>
<td>( A :: B )</td>
<td>Scalar</td>
</tr>
</tbody>
</table>

The Euclidean norm is defined as:

\[
\|m\| = \sqrt{m : m} \tag{I}
\]

A normalization is denoted by an arrow above the respective symbols

\[
\vec{m} = \frac{m}{\|m\|} \tag{II}
\]

[11]

Annex 2: Basic invariants

\[
I_1 = -(\sigma_{11} + \sigma_{22} + \sigma_{33}) \tag{III}
\]

\[
I_2 = \sigma_{12}^2 + \sigma_{13}^2 - \sigma_{11}\sigma_{22} + \sigma_{23}^2 - \sigma_{11}\sigma_{33} - \sigma_{12}\sigma_{13}\sigma_{33} \tag{IV}
\]

\[
I_3 = \sigma_{31}^2\sigma_{22} - 2\sigma_{12}\sigma_{13}\sigma_{33} + \sigma_{11}\sigma_{23}^2 + \sigma_{12}^2\sigma_{33} - \sigma_{11}\sigma_{22}\sigma_{33} \tag{V}
\]

[11]